Vito V. Knoni

CONTRACT REPORT H-68-2

MOTIONS OF SMALL BOATS MOORED IN STANDING WAVES

by

F. Raichlen

August 1968

Sponsored by

Office, Chief of Engineers
U. S. Army

Prepared for

U. S. Army Engineer Waterways Experiment Station
CORPS OF ENGINEERS
Vicksburg, Mississippi

under

Contract No. DA-22-079-civeng-64-11

by

W. M. Keck Laboratory of Hydraulics and Water Resources
Division of Engineering and Applied Science
California Institute of Technology
Pasadena, California

Contract Report H-68-2

MOTIONS OF SMALL BOATS MOORED IN STANDING WAVES

by

Fredric Raichlen

August 1968

Sponsored by

Office, Chief of Engineers, U. S. Army

Prepared for

U. S. Army Engineer Waterways Experiment Station
Corps of Engineers
Vicksburg, Mississippi

Under

Contract DA-22-079-CIVENG-64-11

by

W. M. Keck Laboratory of Hydraulics and Water Resources
Division of Engineering and Applied Science
California Institute of Technology
Pasadena, California

Report No. KH-R-17, August 1968

FOREWORD

The study reported herein was authorized by the Office, Chief of Engineers, in a letter to the U. S. Army Engineer Waterways Experiment Station (WES) dated 2 January 1963. The study was conducted by the W. M. Keck Laboratory of Hydraulics and Water Resources, Division of Engineering and Applied Science, California Institute of Technology, Pasadena, California, during the period August 1967–April 1968. This report was prepared by Dr. Fredric Raichlen.

The work was performed under Contract DA-22-079-CIVENG-64-11. The contract was monitored by Mr. R. Y. Hudson, Chief, Wave Dynamics Branch, Hydraulics Division, WES, under the general supervision of Mr. E. P. Fortson, Jr., Chief, Hydraulics Division. Contracting Officers were COL John R. Oswalt, Jr., CE, and COL Levi A. Brown, CE, Directors of WES.

TABLE OF CONTENTS

TABLE OF CONTENTS (cont'd)

LIST OF FIGURES

LIST OF FIGURES (cont'd)

LIST OF FIGURES (cont'd)

LIST OF FIGURES (cont'd)

LIST OF FIGURES (cont'd)

LIST OF TABLES

ABSTRACT

This study was conducted to determine the dynamic characteristics of small boats moored with non-linear-elastic lines in an asymmetrical manner. The motions being considered are surge motions where the moored boat is allowed to move either in the direction of the bow or the stern, but not in other coordinate directions.

An analytical model is proposed where the small boat is simulated by a block-body which is moored asymmetrically to a fixed dock. A method is developed from which the non-linear restoring forces and the dynamic response of the boat in surge can be obtained. The restoring force which is associated with the boat displacement is defined by the material, condition, and dimensions of the lines and the mooring geometry. From those results, an approximation to the restoring force is made so that a closed solution to the problem is possible. The periods of free oscillation determined by this method are compared to the results of some experiments conducted on a 26-foot boat with a displaced weight of approximately 7000 lbs. The experiments were performed using this small boat moored under different conditions: all lines taut, 4 inches slack in all lines, and 8 inches slack in all lines. These results compared favorably with the analytical results.

The response of seven small boats of various displaced weights were determined analytically to evaluate the range of important wave periods for this sample. The mooring dimensions of these boats were measured in situ and the theoretical approach developed was applied.

The results indicate, for the samples considered, that the important range of periods of forced oscillation for excessive motions of these boats in surge was less than 10 secs. If stiff mooring systems had been employed for all of these boats the important wave period range for these motions could probably be reduced further. Due to the different mooring systems used, the response curves for some of the small boats were highly asymmetrical indicating the possibility of much greater motions in one direction than in another under the action of a periodic symmetrical force.

A limited series of experiments were conducted to determine the effect of the proximity of flotation chambers which are used on some floating slips on the response of the moored boat. It was found that these chambers, as simulated in the laboratory, did not have a significant effect on the dynamic characteristics of the moored boat. However, they did act as floating breakwaters thereby reducing the transmitted wave energy.

1. INTRODUCTION

A moored vessel exposed to waves is an example of a somewhat unique problem in dynamics. In the case of most mechanical systems the forcing function is reasonably well defined and it is the response of this system to the time-varying force which is desired. In the case of a moored vessel the objective of the analysis is the same, i.e., determining the forces induced in the mooring system or the motions of the vessel; however, in that case there is a distinct possibility that the forcing function may also have certain response characteristics which vary significantly in magnitude with wave period. Therefore, even though the major problem which must be solved by the engineer in the design of a harbor is to minimize the motions of the moored boats due to wave action, the dynamics of the harbor itself must also be considered.

The purpose of this report is to consider only the former problem, i.e., the dynamics of the motion of a vessel moored in a standing wave environment; consideration of the wave-induced oscillations of harbors will not be discussed herein. The class of vessels to be considered is small boats of the size usually used for pleasure where it is of interest to predict the important range of wave periods for these pleasure craft moored in a typical manner in a marina. If such a prediction can be made then the problem of minimizing wave-induced oscillations in a small boat harbor may be simplified because it would be sufficient to investigate the response of a harbor to waves only over the limited range of wave periods which affects small craft.

In a previous report on wave-induced oscillations of small boats (Raichlen, 1965) some attention was given to a review of literature dealing with both the dynamics of moored bodies and the evaluation of the added hydrodynamic mass. In this report these articles will not be reviewed but a list of these references is included in a supplemental bibliography.

It is first important to consider some of the major differences between small and large boats which relate to their dynamics. Of course, the most obvious difference between the two cases is size; in the case of small pleasure boats the vessels which are of interest are less than approximately 60 feet long and the displaced weights are less than approximately 10 tons as compared to large vessels whose lengths may be 300 feet to 700 feet with displaced weights ranging from 9,000 to 50,000 tons. However, it is not the displaced weight alone which is of importance but, as will be shown in a later section of this report, it is really the ratio of the restoring force caused by the mooring system to the inertial force (which depends on the displaced weight of the vessel) which is important in defining the response of the vessel to wave excitation.

With respect to the ratio of restoring forces to inertial forces the nature of the restoring force for large ships can be quite different from that for small boats. Wilson (1967a) has described in detail the mooring systems used by a number of large vessels. In general those systems consisted of a large number of lines extending from the bow, the stern, and the mid-ship of the vessel to the dock which restrict motion both in a fore and aft direction as well as in a direction perpendicular to the dock. For example, for ships ranging in weight from 10,000 to 60,000 tons Wilson (1967a) found that the total number of lines used in mooring may range from 15 to 40 respectively. Therefore, even though the elastic characteristics of the individual mooring lines may be quite different from one another, on the average the restoring force for motion in the fore and aft direction for similar displacements would be approximately the same. (It should be noted, however, that for motions of a large vessel in sway, i.e., in a direction perpendicular to the dock, the restoring forces would be highly asymmetrical.) In addition to the mooring features mentioned, usually the mooring lines for large ships have slack to allow for motions due to changes in the tide. It will be shown in the analysis that the introduction of even a minor amount of slack can affect the restoring forces and thus the response characteristics of small moored vessel significantly.

On the other hand, in some respects the mooring systems for small boats are quite different from those of large vessels. A photograph of a small boat moored in a typical fashion to a floating slip is presented in Fig. 1.1. This photograph shows that for this case only a few lines are being used; there are only two bow lines and two stern lines. Under these conditions, it is evident that the restoring force for motion of the vessel in one direction may be quite different from the restoring force for similar displacements in the opposite direction. From observations of other mooring arrangements, it was evident from this study that there is less probability of having symmetrical forces in the surge direction for small boats than for large ships. This asymmetry can have a significant effect upon the motions of the

Fig. 1.1. Photograph of Moored Small Boat

boat and potential boat damage. For instance, if a small boat is moored in a slip with little clearance between the bow of the boat and the front of the slip, impact damage to the bow may be possible due to the asymmetrical restoring forces. This type of damage possibly could be eliminated by using a stiffer more symmetrical mooring system. An additional effect of asymmetrical moorings is that particular boat fittings may be stressed an excessive amount. Indeed it may be possible that the fittings would fail before the lines part. Conversely, in the case of large vessels damage criteria resulting from ship-mooring dynamics have been based primarily upon the parting of the mooring lines (see Wilson, 1967a).

An interesting problem is introduced by the economics of mooring of small boats which may have direct effect upon damage by impact. Since the rent a boat owner generally pays depends on the slip length, there is a tendency for the owner to try to minimize the length of the slip and moor his boat in the smallest slip available. This, of course, will result in minimizing the clearance between the bow of the boat and the front of the slip thereby increasing the possibility of potential impact damage under asymmetrical mooring conditions.

Although only a few examples of the differences between large and small boat mooring considerations have been presented in this discussion, through these it is evident that attention must be given to some of the details of mooring for small boats which could perhaps be neglected for large vessels.

The objective of this report is to discuss in detail some of the aspects of the asymmetrical restoring forces of small boats when moored with elastic non-linear restraints. In connection with this analysis of mooring dynamics experiments were conducted to determine the periods of the free oscillations of a 26-foot boat moored in various ways to a floating slip. The analysis presented herein has been applied to a number of small boats whose in situ mooring dimensions were obtained in order to determine the important range of wave periods with respect to surge motions for a reasonably wide selection of boat sizes and mooring conditions. The method of approach will be covered in sufficient detail such that additional information can be obtained by other investigators to more firmly define the above-mentioned period range.

2. THEORETICAL CONSIDERATIONS

In this section an analysis will be presented which describes the motions of a moored body in surge when exposed to a standing wave system. The mooring system used in the analytical model can result in non-linear asymmetrical restraining forces which resist the wave-induced motions. Only surge motions (boat displacements either toward the bow or toward the stern) are considered, and the boat is treated as a block body with no attempt being made to fully describe in detail the shape of the vessel. This block body approach is probably more applicable for inboard power boats rather than sail boats due to the relatively large draft-to-beam of the latter. The justification of these assumptions will be shown in the comparison of the theory to experimental results in Section 4. This analysis follows from those reported by Wilson (1958), Kilner (1960), and Raichlen (1965), and, therefore, the initial portion dealing with the development of the equation of motion of a moored vessel in surge will be presented in condensed manner. (For a more detailed presentation the interested reader is referred to Raichlen (1965).) The innovation in the following, which will be presented in detail, concerns the incorporation in the analytical model of a non-linear asymmetrical restoring force.

2.1 Equation of Motion in Surge

As mentioned, in order to investigate some of the fundamental aspects of the mooring of small boats the dynamics of motion of a block body moored in a standing wave environment is studied. The body under discussion is a rectangular parallelpiped of length 2L, beam B, and draft D moored in a way such that the only allowable motions are surge

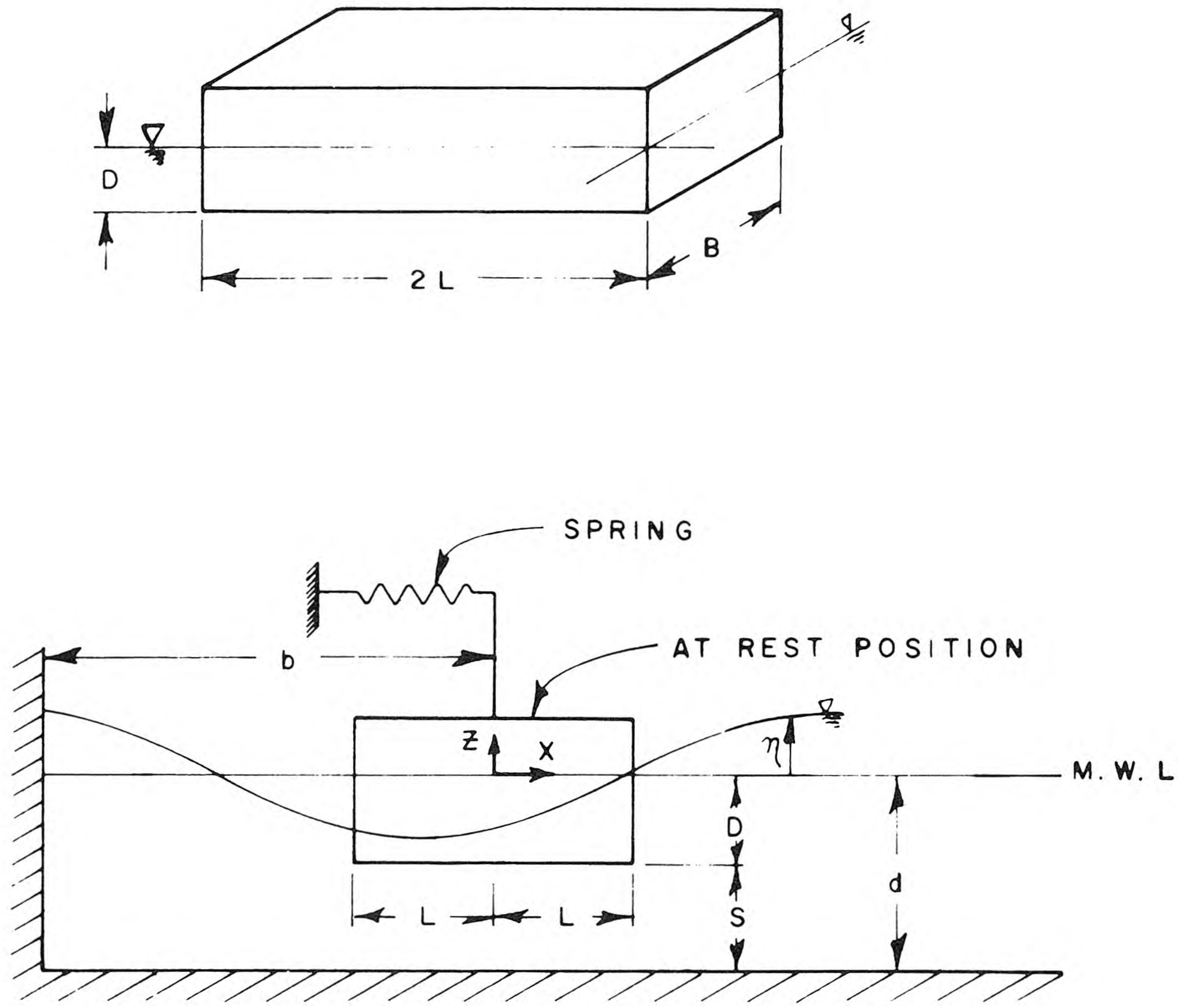

Fig. 2.1. Schematic Diagram of Block Body

motions. A schematic diagram of this body moored in a standing wave system is shown in Fig. 2.1. The standing wave is formed in water of a constant depth d by a progressive wave which is reflected from a perfectly reflecting surface located a distance b from the center of the moored body. The x-coordinate is measured from the center of the body in the at-rest-position and denotes the movement of the center in surge motion.

Using small amplitude wave theory the wave amplitude η, velocity potential Φ, and the water particle velocity u of the standing wave are described by the following three expressions:

$$\eta = A \cos k(b+x) \cos \sigma t \tag{2.1}$$

$$\Phi = \frac{Ag}{\sigma} \frac{\cosh k(d+z)}{\cosh kd} \cos k(b+x) \sin \sigma t \tag{2.2}$$

$$u = \frac{Agk}{\sigma} \frac{\cosh k(d+z)}{\cosh kd} \sin k(b+x) \sin \sigma t \tag{2.3}$$

where A is the standing wave amplitude, k is the wave number (2π/wave length, λ), σ is the circular wave frequency (2π/wave period, T) g is the acceleration of gravity, and the other quantities are described in the definition sketch, Fig. 2.1.

The equation of motion in surge of a moored body is:

$$M\ddot{x} = \sum (\text{External Forces})$$

$$M\ddot{x} = F_p + F_i + F_d + F_r \tag{2.4}$$

where M is the mass of the body, $\ddot{x}$ is the acceleration of the body $(\frac{d^2x}{dt^2})$, F_p is a driving force due to pressure, F_i is an inertial force, F_d is a drag force, and F_r is a restoring force due to the mooring system.

The pressure force is due to the pressures acting on the ends of the block body, and it is given by:

$$F_p = B \int_{-D}^{\eta} p(-L, z)\, dz - B \int_{-D}^{\eta} p(+L, z)\, dz \tag{2.5}$$

where p (-L, z) and p (+L, z) denote the pressure distributions on the two ends of the body due to the standing wave. From the equation of motion of the fluid and the velocity potential described by Eq. 2.2 the pressure distribution under a standing wave is obtained in the form:

$$p = \gamma \eta \frac{\cosh k\,(z+d)}{\cosh kd} - \gamma z \tag{2.6}$$

Substituting Eq. 2.6 into Eq. 2.5 the net force per unit width acting on the body is:

$$\frac{F_p}{B} = \gamma\,(\eta_{-L} - \eta_{+L}) \left[\frac{\sinh kd - \sinh ks}{k \cosh kd} \right] - \frac{\gamma}{2}\,(\eta^2{}_{-L} - \eta^2{}_{+L}) \tag{2.7}$$

and substituting the appropriate expressions into Eq. 2.7 one obtains:

$$\frac{F_p}{B} = 2\gamma A \left[\frac{\sinh kd - \sinh ks}{k \cosh kd} \right] \sin kL \sin kb \cos \sigma t$$

$$- \gamma \frac{A^2}{2} \sin 2kL \sin 2kb \cos^2 \sigma\, t \tag{2.8}$$

For small amplitude waves Eq. 2.8 can be simplified further since the ratio of the second term in Eq. 2.8 to the first term is of the order of the wave steepness, $2A/\lambda$. Therefore, neglecting this term, Eq. 2.8 becomes:

$$\frac{F_p}{B} = 2\gamma A \left[\frac{\sinh kd - \sinh ks}{k \cosh kd} \right] \sin kL \sin kB \cos \sigma t \tag{2.9}$$

An interesting simplification has been introduced by Wilson (1958) which will be used in the derivation of all terms (except the restoring force) in Eq. 2.4. This is the concept of the volume average of water particle velocities and accelerations defined as:

$$U = \frac{1}{2LD} \int_{-L}^{L} \int_{-D}^{\eta} u \, dx \, dz \tag{2.10a}$$

$$\dot{U} = \frac{1}{2LD} \int_{-L}^{L} \int_{-D}^{\eta} \dot{u} \, dx \, dz \tag{2.10b}$$

Substituting Eq. 2.3 and its time derivative into Eq. 2.10 these volume averages become:

$$U = \frac{Ag}{LD\sigma} \left[\frac{\sinh kd - \sinh ks}{k \cosh kd} \right] \sin kL \sin kb \sin \sigma t \tag{2.11a}$$

$$\dot{U} = \frac{Ag}{LD} \left[\frac{\sinh kd - \sinh ks}{k \cosh kd} \right] \sin kL \sin kb \cos \sigma t \tag{2.11b}$$

Therefore, Eq. 2.9 can be further simplified by the substitution of Eq. 2.11b:

$$F_p = M \dot{U} \tag{2.12}$$

where $M = 2\rho LBD$, the mass of the fluid displaced by the body. Hence, Eq. 2.12 attributes the driving force due to the net pressure acting on the ends of the block body, as a first approximation, to the water particle accelerations in the standing wave averaged over the volume displaced by the body.

The inertial force in Eq. 2.4, F_i, is introduced by the unsteady nature of the problem, i.e., as the body accelerates or decelerates its motion affects the surrounding fluid. For instance, if the fluid were at rest and the body were accelerated there would be a force opposing motion, other than viscous forces, caused by the body accelerating a portion of the surrounding fluid. This added force is usually represented as the product of the acceleration and an added hydrodynamic mass, M'. For the case of the unsteady motion of a body in a fluid whose motion is unsteady this force still exists, but the acceleration or deceleration is relative to the fluid particle acceleration or declaration. The assumption is made in this analysis that the important relative acceleration is between the acceleration of the center of the body and the volume average of the water particle acceleration. Therefore, the inertial force is expressed as:

$$F_i = M'_x(\dot{U} - \ddot{x}) \tag{2.13}$$

where M'_x is the added hydrodynamic mass of the body in surge.

In a similar manner a viscous drag force can be defined in terms of a relative velocity as:

$$F_d = \frac{\rho}{2} C_{D_x} B D (U - \dot{x}) \left| U - \dot{x} \right| \tag{2.14}$$

where C_{D_x} is a drag coefficient for the body in surge. This assumes that the important water particle velocity which is related to the viscous effects is the average of the water particle velocity over the displaced volume of the body.

Substituting Eqs. 2.12, 2.13 and 2.14 into Eq. 2.4 the following expression is obtained for the equation of motion of the body in surge:

$$M\ddot{x} = M\dot{U} + M'_x(\dot{U} - \ddot{x}) + \frac{\rho}{2} C_{D_x} BD(U - \dot{x})\left|U - \dot{x}\right| - F_r \qquad (2.15)$$

In Eq. 2.15 the restraining force due to the mooring system has not been defined and is shown with a minus sign to indicate it acts opposite to the direction of motion.

To simplify the problem the effect of viscous damping is neglected in Eq. 2.15. As far as the investigation of some fundamental aspects of the problem is concerned neglecting damping is considered to be reasonable, since the range of the important wave periods in the mooring dynamics (which is one of the major objectives of this study) is affected little by damping.

With these considerations in mind, Eq. 2.15 can be simplified further and rearranged to yield:

$$\ddot{x} + \frac{F_r}{C_M M} = \dot{U} \qquad (2.16)$$

where the virtual mass coefficient, C_M, is defined as $C_M = 1 + \frac{M'_x}{M}$. The volume average quantities U and $\dot{U}$ can be rewritten as:

$$U = \zeta \sin \sigma t \qquad (2.17a)$$

$$\dot{U} = \zeta \sigma \cos \sigma t \qquad (2.17b)$$

where $$\zeta = \frac{Ag}{LD\sigma}\left[\frac{\sinh kd - \sinh ks}{k \cosh kd}\right] \sin kL \sin kb \qquad (2.17c)$$

Therefore, the equation of motion in the x-direction (Eq. 2.16) becomes:

$$\ddot{x} + \frac{F_r}{C_M M} = \zeta \sigma \cos \sigma t \qquad (2.18)$$

The problems in solving Eq. 2.18 arise primarily due to the complex form of the restoring force relation. The variation of the virtual mass coefficient C_M with boat shape and other conditions is not well known, but it will be shown that the effect of this quantity on the dynamic response of a small moored boat in most cases is not as important as the effect of the restoring force.

A number of possible restoring force relations are shown in a schematic way in Fig. 2.2. The simplest system presented is the linear-symmetrical system which provides for the same linear springs restraining the motion of the floating body in the fore and the aft direction. The more general case for the linear spring system is the bi-linear asymmetrical system shown in Fig. 2.2b which provides linear restraint of different stiffness in the two directions. Either one of these systems can degenerate into a non-linear restraint by allowing for free travel of the floating body before a restoring force is developed. This type of restraint is presented in Fig. 2.2c. From these three systems the next logical extension is to non-linear restoring forces. The last sequence of restoring forces shown in Figs. 2.2d and 2.2e for the non-linear cases have their analogies in the linear systems which have just been described.

2.1.1. Linear Symmetrical Restoring Force

The case which has been studied previously and reported in detail by Raichlen (1965) is the system of linear-symmetrical restoring forces. Even though this type of mooring system is not representative of those found in small boat harbors, except for small motions, the results of the analysis are extremely instructive and

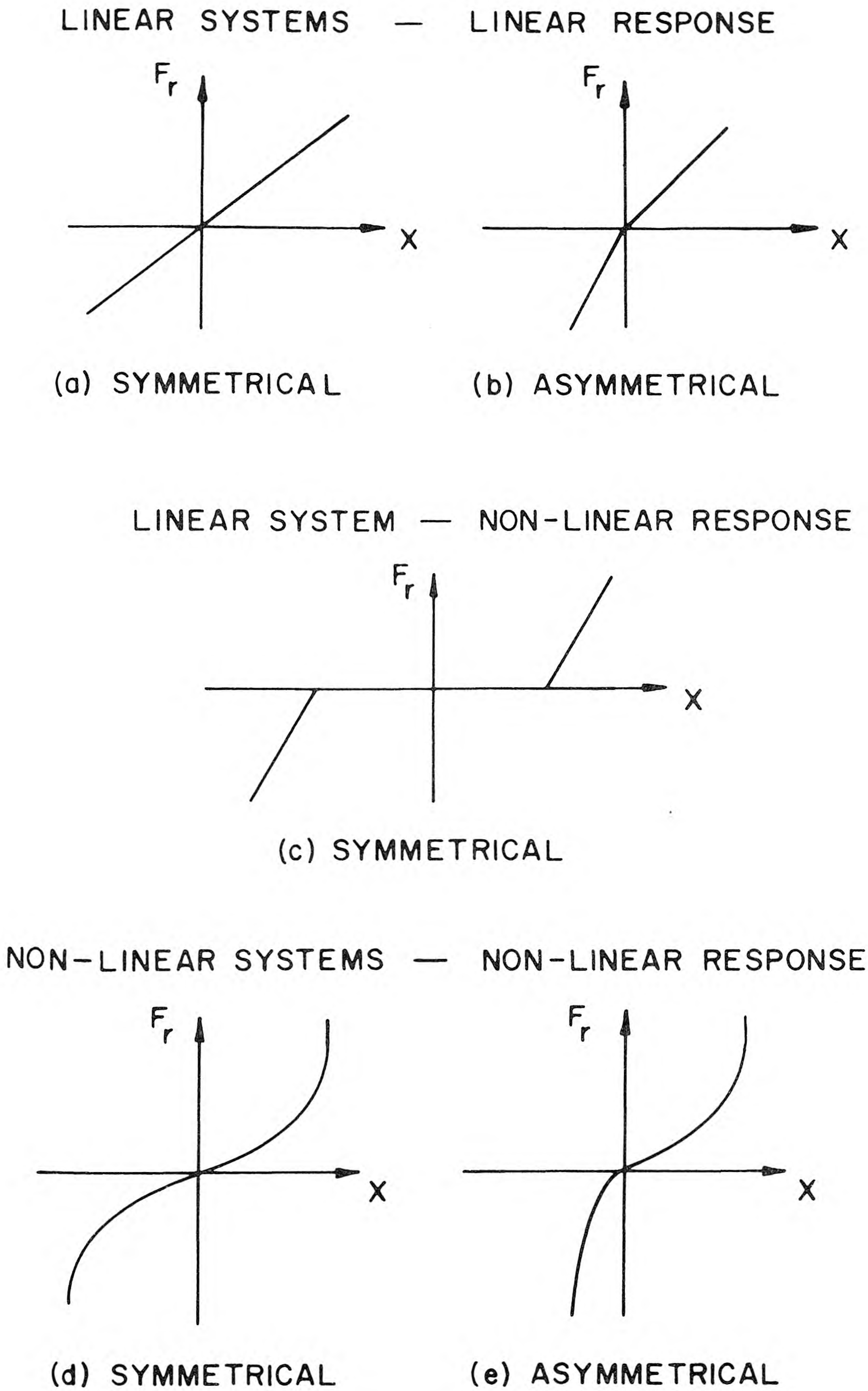

Fig. 2.2. Schematic Diagram of Restoring Force Relations

answer certain questions relating to the mooring dynamics which might be disguised by the complications introduced by non-linear restraints. One major question that was answered by an analytical and experimental program using a body moored in a linear manner is the justification of using volume average quantities such as shown in Eqs. 2.10 and 2.11 to describe the forcing function in the equation of motion.

Consider a moored body restrained by a linear-symmetrical restoring force described by:

$$F_r = Kx \tag{2.19}$$

Eq. 2.18 can then be rewritten as:

$$\ddot{x} + \omega_n^2 x = \zeta \sigma \cos \sigma t \tag{2.20}$$

where $$\omega_n^2 = \frac{K}{C_M M}$$

Assuming a solution of the form $x = X \cos(\sigma t - \varphi)$ where φ is a phase angle it can be shown that a solution to Eq. 2.20 has the form:

$$\sigma^2 + \frac{\zeta}{X}\sigma - \omega_n^2 = 0 \tag{2.21}$$

with $\varphi = 0, \pi$.

The easily recognized solution to the amplitude response of this simple undamped linear system can be obtained from Eq. 2.21; rearranging that expression:

$$\frac{X\sigma}{\zeta} = \frac{1}{\dfrac{\omega_n^2}{\sigma^2} - 1} \tag{2.22}$$

To determine the adequacy of Eq. 2.22, or more importantly the correctness of the forcing function ζ in Eq. 2.22, experiments were conducted in the laboratory using a neutrally buoyant rectangular parallelepiped restrained by a linear spring system. These experiments have been fully described by Raichlen (1965) and only one example of the results will be described here before proceeding to discuss more complicated restoring forces.

The theoretical response curve of a linearly moored body is presented in Fig. 2.3 along with experimental results obtained in the laboratory for the specified conditions shown on the figure. The abscissa is the ratio of the natural period of the body in surge to the wave period, i.e., the ratio of the forcing frequency to the natural frequency, and the ordinate is the ratio of the maximum surge motion of the body to the standing wave amplitude A. It is seen that the agreement of the theoretical curve described by Eq. 2.21 or Eq. 2.22 is relatively good over most of the range of wave periods tested. An example of the variation of the forcing function ζ with wave period (described by Eq. 2.17c) is the zeroes of the response curves shown in Fig. 2.3 which are imposed by the trigonometric terms in Eq. 2.17c. The effect and physical significance of these zeroes of response have been fully discussed in Raichlen (1965) and will not be pursued in this section. It was concluded from these laboratory studies that the response of a neutrally buoyant body moored with linear symmetrical springs was adequately described by Eqs. 2.20, 2.21, and 2.22 over a wide range of ratios of depth to wave length and body length to wave length. For this reason it is felt that the assumptions made in developing

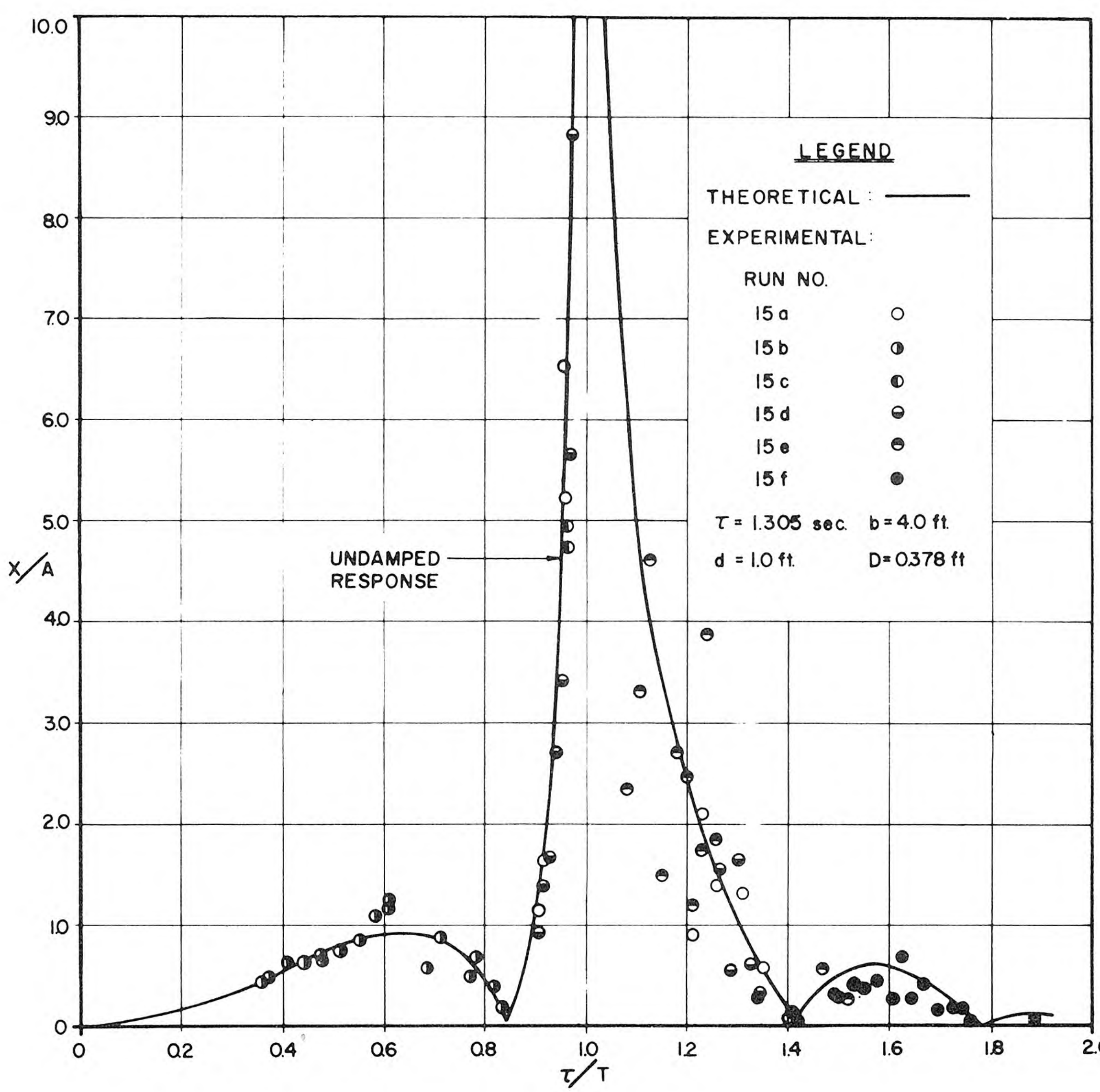

Fig. 2.3. Response Curve of Body Moored with Linear Springs (after Fig. 2.2a).

the equation of motion described by Eq. 2.18 are reasonable and in particular the description of the driving forces acting on a moored body in a standing wave environment are adequately described by the volume average quantities, U and $\dot{U}$ presented in Eqs. 2.10a and 2.10b and the equations which result from the subsequent application of these expressions.

2.1.2. Non-Linear Asymmetrical Restoring Force

In lieu of solving the equation of motion (Eq. 2.18) for the different linear restoring force relations shown in Fig. 2.2 the case of a non-linear asymmetrical restoring force will be treated. Due to certain approximations used, which will be fully described in this section, the cases of bi-linear asymmetrical restoring forces and linear symmetrical systems with free travel can be obtained directly from the results of this more general treatment.

Consider the equation of motion of the moored body in surge described by Eq. 2.18 and the restoring force F_r described by the following expressions:

$$F_r(x) = F_1(x) \text{ for } x > 0 \tag{2.23a}$$

$$F_r(x) = F_2(x) \text{ for } x < 0 \tag{2.23b}$$

where x is defined as being positive for motion toward the bow and negative for motion toward the stern. (The functions represented in Eq. 2.23 may take any form: linear or non-linear, symmetrical or asymmetrical, free travel or no free travel.)

For simplicity of solution, the equation of motion (Eq. 2.18) is rewritten so that the phase relation between the forcing function and the body response is incorporated in the forcing function. This will not affect the general nature of the solution, since the importance of the phase angle is simply to describe the lag or lead of the motion relative to the forcing function and this is still preserved in the solution. Hence, Eq. 2.18 is rewritten as:

$$\ddot{x} + \frac{F_r}{C_M M} = \zeta \sigma \cos(\theta - \varphi) \qquad (2.24)$$

where $\theta = \sigma t$ and φ is a phase angle and with the conditions of Eqs. 2.23a and 2.23b describing the variation of F_r.

If the restoring forces $F_1(x)$ and $F_2(x)$ are different then it is reasonable to assume that the mean position of motion of the vessel will be different from the at-rest-position of the vessel. This would occur because one set of mooring lines (either bow or stern) is stiffer than the other. Therefore, the general solution to Eq. 2.24 is taken as:

$$x = \delta + X \cos \theta \qquad (2.25)$$

A solution based on Eq. 2.25 is somewhat approximate since it neglects harmonics other than the fundamental. However, it is felt that this is justified since for symmetric mooring forces having the form of a power of x as high as 7 it has been found (see Kilner (1960)) that the significant harmonic was less than 10% of the fundamental. The motion described by Eq. 2.25 is shown in a schematic manner in Fig. 2.4 along with assumed restoring forces described by Eqs. 2.23a and 2.23b. By setting x = 0, Eq. 2.25 describes the degree of asymmetry of the restoring force, that is:

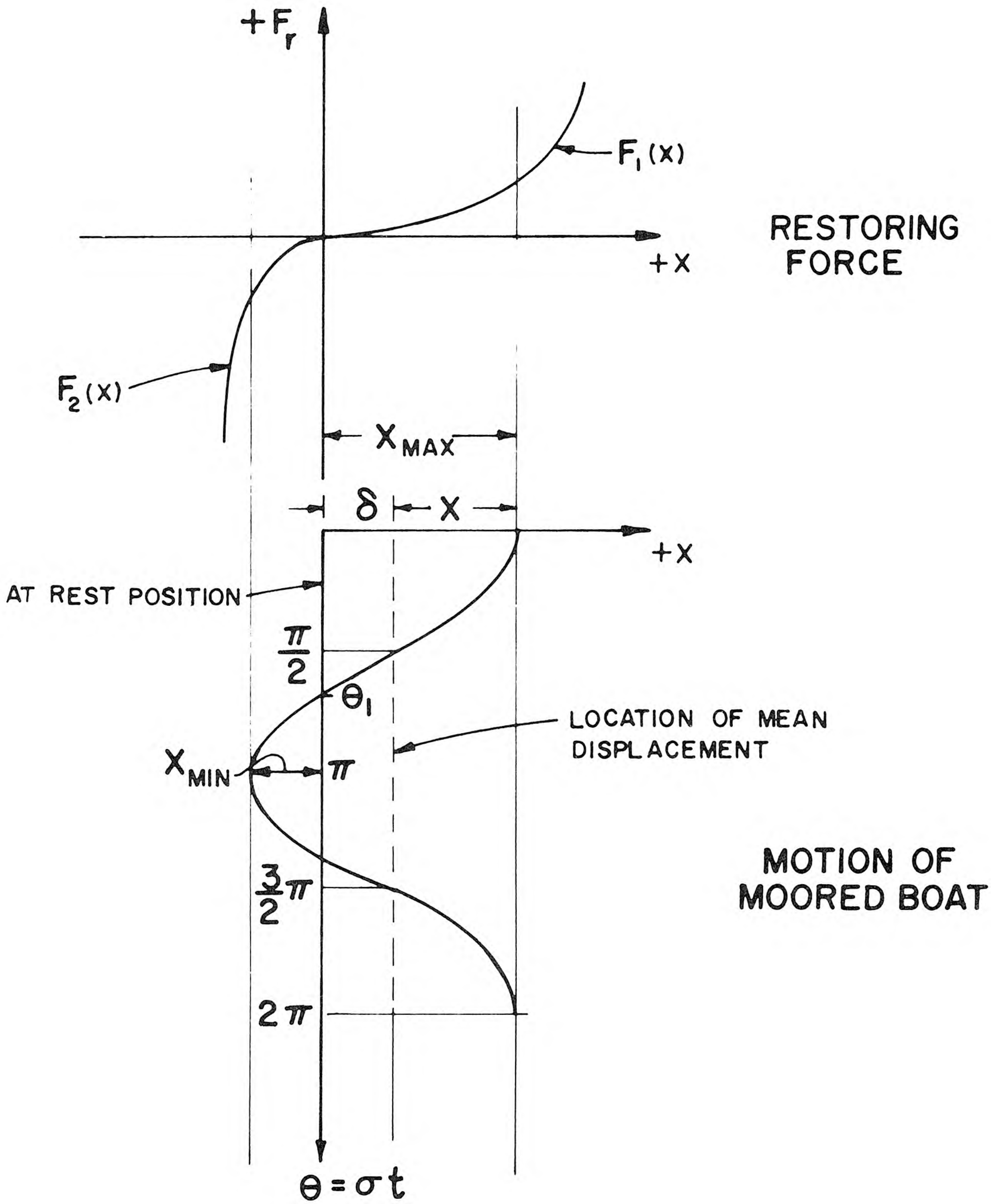

Fig. 2.4. Schematic Diagram of Non-Linear Asymmetrical Restoring Force and Resultant Boat Motion

$$\delta = -X \cos \theta_1 \tag{2.26}$$

and for $\theta_1 = \pi/2$ the displacement of the mean position from the at-rest position, δ, becomes equal to zero. It is noted from Fig. 2.4 that this implies that $F_1(x) = - F_2(x)$ and the restoring force and the resultant motion are symmetrical about the at-rest-position of the boat. If, on the other hand, $\theta_1 = \pi$ the boat motion is highly asymmetrical and in accordance with Eqs. 2.26 and 2.25 there would be no motion of the vessel in the minus x-direction. This is equivalent to saying that the mooring lines which result in the restoring force $F_2(x)$ are infinitely stiff compared to those which provide a restoring force $F_1(x)$.

Klotter (1951) has described the solution of an equation such as Eq. 2.24 with non-linear restoring forces based on the averaging method of W. Ritz. Rewriting Eq. 2.24 as:

$$E(x) = \ddot{x} + \frac{F_r}{C_M M} - \zeta \sigma \cos (\theta - \varphi) \tag{2.27}$$

The averaging method furnishes the following two conditions:

$$\int_o^{2\pi} E(x) \cos \theta \, d\theta = 0 \tag{2.28a}$$

$$\int_o^{2\pi} E(x) \sin \theta \, d\theta = 0 \tag{2.28b}$$

Substituting Eq. 2.27 into Eqs. 2.28a and 2.28b the two conditions to be solved become:

$$\int_o^{2\pi} \left[\ddot{x} + \frac{F_r}{C_M M} - \zeta \sigma \cos (\theta - \varphi) \right] \cos \theta \, d\theta = 0 \tag{2.29a}$$

$$\int_0^{2\pi} \left[\ddot{x} + \frac{F_r}{C_M M} - \zeta \sigma \cos(\theta - \varphi) \right] \sin\theta \, d\theta = 0 \qquad (2.29b)$$

Consider the first and third terms in Eq. 2.29a along with the assumed form of the solution, Eq. 2.25. The first term becomes:

$$\int_0^{2\pi} \ddot{x} \cos\theta \, d\theta = \int_0^{2\pi} - X \sigma^2 \cos^2\theta \, d\theta = - \pi X \sigma^2 \qquad (2.30)$$

and the third term becomes

$$\int_0^{2\pi} \zeta \sigma \cos(\theta - \varphi) \cos\theta \, d\theta = \int_0^{2\pi} \zeta \sigma \cos^2\theta \cos\varphi \, d\theta = \pi \zeta \sigma \cos\varphi \qquad (2.31)$$

The integral which is the second term in Eq. 2.29a, referring to Fig. 2.4, can be integrated over three intervals; hence:

$$\int_0^{2\pi} \frac{F_r}{C_M M} \cos\theta \, d\theta = \int_0^{\theta_1} \frac{F_1(x)}{C_M M} \cos\theta \, d\theta + \int_{\theta_1}^{2\pi - \theta_1} \frac{F_2(x)}{C_M M} \cos\theta \, d\theta$$

$$+ \int_{2\pi - \theta_1}^{2\pi} \frac{F_1(x)}{C_M M} \cos\theta \, d\theta \qquad (2.32)$$

In order to solve Eq. 2.32 so that the solution to Eq. 2.29a can be obtained it is necessary to evaluate the restoring forces $F_1(x)$ and $F_2(x)$ for a particular system of mooring lines. To carry out the indicated integrations it is assumed that the functions $F_1(x)$ and $F_2(x)$ can be represented reasonably well over the range of displacements which are of interest by polynomials consisting of terms which are odd powers of x. Therefore, the following approximations are used:

$$F_1(x) = a_1 x + a_2 x^3 + a_3 x^5 \qquad (2.33a)$$

$$F_2(x) = r_1 x + r_2 x^3 + r_3 x^5 \qquad (2.33b)$$

The odd powers of x are used so that the restoring force reverses sign correctly in the equation of motion. It will be shown that for most cases

Eqs. 2.33a and 2.33b are good approximations to the restoring force and fit the restoring force particularly well when small boats are moored with slack lines. The solution to Eq. 2.32 is obtained by substituting Eq. 2.25 into Eqs. 2.33a and 2.33b and then introducing the resulting expressions into Eq. 2.32. After integration Eq. 2.32 (without the constant factor $C_M M$) becomes:

$$
\begin{aligned}
\int_0^{2\pi} & F_r \cos\theta \, d\theta \\
&= \left\{ r_1 X + \left[3\left(\frac{\delta}{X}\right)^2 + \frac{3}{4} \right] r_2 X^3 + \left[5\left(\frac{\delta}{X}\right)^4 + \frac{15}{2}\left(\frac{\delta}{X}\right)^2 + \frac{5}{8} \right] r_3 X^5 \right\} \pi \\
&\quad + \left\{ (a_1 - r_1) X + \left[3\left(\frac{\delta}{X}\right)^2 + \frac{3}{4} \right] (a_2 - r_2) X^3 \right. \\
&\qquad \left. + \left[5\left(\frac{\delta}{X}\right)^4 + \frac{15}{2}\left(\frac{\delta}{X}\right)^2 + \frac{5}{8} \right] (a_3 - r_3) X^5 \right\} \theta_1 \\
&\quad + \left\{ (a_1 - r_1)\left(\frac{\delta}{X}\right) X + \left[\left(\frac{\delta}{X}\right)^3 + \frac{9}{4}\left(\frac{\delta}{X}\right) \right] (a_2 - r_2) X^3 \right. \\
&\qquad \left. + \left[\left(\frac{\delta}{X}\right)^5 + \frac{15}{2}\left(\frac{\delta}{X}\right)^3 + \frac{25}{8}\left(\frac{\delta}{X}\right) \right] (a_3 - r_3) X^5 \right\} 2 \sin\theta_1 \\
&\quad + \left\{ \frac{(a_1 - r_1)}{4} X + \left[\left(\frac{3}{4}\,\frac{\delta}{X}\right)^2 + \frac{1}{4} \right] (a_2 - r_2) X^3 \right. \\
&\qquad \left. + \left[\frac{5}{4}\left(\frac{\delta}{X}\right)^4 + \frac{5}{2}\left(\frac{\delta}{X}\right)^2 + \frac{15}{64} \right] (a_3 - r_3) X^5 \right\} 2 \sin 2\theta_1 \\
&\quad + \left\{ \left(\frac{1}{4}\right)\left(\frac{\delta}{X}\right)(a_2 - r_2) X^3 + \left[\frac{5}{6}\left(\frac{\delta}{X}\right)^3 + \frac{25}{48}\left(\frac{\delta}{X}\right) \right] (a_3 - r_3) X^5 \right\} 2 \sin 3\theta_1 \\
&\quad + \left\{ \frac{1}{32}(a_2 - r_2) X^3 + \left[\frac{5}{16}\left(\frac{\delta}{X}\right)^2 + \frac{3}{64} \right] (a_3 - r_3) X^5 \right\} 2 \sin 4\theta_1 \\
&\quad + \left\{ \frac{1}{16}\,\frac{\delta}{X}(a_3 - r_3) X^5 \right\} 2 \sin 5\theta_1 \\
&\quad + \left\{ \frac{1}{192}(a_3 - r_3) X^5 \right\} 2 \sin 6\theta_1
\end{aligned}
\tag{2.34}
$$

The ratio $\frac{\delta}{X}$ which appears in Eq. 2.34 can be replaced by $-\cos\theta_1$ (see Eq. 2.26). Therefore, before Eq. 2.34 can be solved for a particular value of X, the angle of zero crossing, θ_1, must be obtained. An expression which can be solved for θ_1 as a function of X can be obtained by averaging the equation of motion (Eq. 2.24) over one wave period. When this is done only one term remains:

$$\int_o^{2\pi} \frac{F_r}{C_M M}\, d\theta = 0 \tag{2.35a}$$

or:
$$\int_o^{\theta_1} F_1(x)d\theta + \int_{\theta_1}^{2\pi-\theta_1} F_2(x)\, d\theta + \int_{2\pi-\theta_1}^{2\pi} F_1(x)d\theta = 0 \tag{2.35b}$$

After substituting Eqs. 2.33, 2.25, and 2.26 into Eq. 2.35b and performing the indicated integrations the following equation is obtained in terms of X and θ_1.

$$\begin{aligned}
&\Big\{-r_1 X \cos\theta_1 - \Big[\cos^3\theta_1 + \tfrac{3}{2}\cos\theta_1\Big] r_2 X^3 \\
&\quad - \Big[\cos^5\theta_1 + 5\cos^3\theta_1 + \tfrac{15}{8}\cos\theta_1\Big] r_3 X^5\Big\}\,\pi \\
&+ \Big\{(r_1 - a_1) X \cos\theta_1 + \Big[\cos^3\theta_1 + \tfrac{3}{2}\cos\theta_1\Big](r_2 - a_2) X^3 \\
&\quad + \Big[\cos^5\theta_1 + 5\cos^3\theta_1 + \tfrac{15}{8}\cos\theta_1\Big](r_3 - a_3) X^5\Big\}\,\theta_1 \\
&+ \Big\{(a_1 - r_1) X + \Big[3\cos^2\theta_1 + \tfrac{3}{4}\Big](a_2 - r_2) X^3 \\
&\quad + \Big[5\cos^4\theta_1 + \tfrac{15}{2}\cos^2\theta_1 + \tfrac{5}{8}\Big](a_3 - r_3) X^5\Big\} \sin\theta_1 \\
&+ \Big\{\Big[\tfrac{3}{4}\cos\theta_1\Big](r_2 - a_2) X^3 + \Big[\tfrac{5}{2}\cos^3\theta_1 + \tfrac{5}{4}\cos\theta_1\Big](r_3 - a_3) X^5\Big\} \sin 2\theta_1 \\
&+ \Big\{\tfrac{1}{12}(a_2 - r_2) X^3 + \Big[\tfrac{5}{6}\cos^2\theta_1 + \tfrac{5}{48}\Big](a_3 - r_3) X^5\Big\} \sin 3\theta_1 \\
&+ \Big\{\Big[\tfrac{5}{32}\cos\theta_1\Big](r_3 - a_3) X^5\Big\} \sin 4\theta_1 \\
&+ \Big\{\tfrac{1}{80}(a_3 - r_3) X^5\Big\} \sin 5\theta_1 = 0
\end{aligned} \tag{2.36}$$

Hence, for a particular mooring configuration, i. e., given values of the coefficients a_1, a_2, a_3 and r_1, r_2, r_3, the variation of θ_1 with X can be determined from Eq. 2.36. From this variation Eq. 2.34 can then be evaluated.

Eq. 2.29b can be solved in a similar manner. In the evaluation of the integrals in this equation it can be shown that the first and the second terms are identically equal to zero, and hence, Eq. 2.29b becomes:

$$\int_0^{2\pi} \zeta\,\sigma \cos(\theta - \varphi) \sin\theta \, d\theta = \zeta\,\sigma \sin\varphi = 0 \qquad (2.37)$$

Therefore: $\varphi = 0, \pi$; which is as it should be for the forced oscillations of an undamped dynamic system. The response is in phase with the forcing function to one side of resonance and 180^{o} out of phase with the forcing function to the other side of resonance.

Substituting Eqs. 2.30 and 2.31 into Eq. 2.29a the following general expression is obtained which describes the response in surge of the arbitrarily moored body in terms of its maximum displacement in the positive x-direction from the at-rest-position:

$$\sigma^2 + \frac{\zeta(1 - \cos\theta_1)}{X_{max}}\,\sigma - \frac{(1 - \cos\theta_1)}{\pi X_{max} C_M M} \int_o^{2\pi} F_r \cos\theta \, d\theta = 0 \qquad (2.38)$$

where $X_{max} = \delta + X$

$\cos\theta_1 = -\dfrac{\delta}{X}$

and $X_{max} = X(1 - \cos\theta_1)$

The integral in Eq. 2.38 is given by Eq. 2.34 with the variation of θ_1 with X given by Eq. 2.36. It should be noted, the most straightforward solution to Eq. 2.38 is to solve it for σ given values of ζ, X, and θ_1. Since the maximum displacement in the negative x-direction, X_{min}, can be obtained from the relation: $X_{min} = 2\delta - X_{max}$, the complete response curve is fully defined.

Eq. 2.38 and the equations it depends upon are difficult to interpret in this form for the case of the response of a body with non-linear asymmetrical restoring forces. Therefore, certain special cases which result from simplifying Eq. 2.38 will be discussed in detail before proceeding with the more general discussion of small-boat mooring problems.

As a first case the simple problem of a linear-symmetrical restoring force is considered. For this system the approximate expressions for the restoring force (Eqs. 2.33a and 2.33b) are simplified since $a_1 = r_1$ and $a_2 = a_3 = r_2 = r_3 = 0$. Therefore, Eq. 2.36 becomes: $\pi r_1 X \cos \theta_1 = 0$ or $\theta_1 = \pi/2$. This is correct, since the problem under consideration has symmetrical restoring forces. With this restoring force, Eq. 2.34 becomes:

$$\int_0^{2\pi} F_r \cos \theta \, d\theta = \pi r_1 X. \tag{2.39}$$

Substituting Eq. 2.39 into Eq. 2.38 that expression reduces to Eq. 2.21. (This, to some extent, serves as a check of the computational procedure.)

The case of the bilinear asymmetrical restoring force shown in Fig. 2.2.b is interesting because it represents a first approximation to the analogous non-linear problem. In this case $a_1 \neq r_1$ and as before $a_2 = a_3 = r_2 = r_3 = 0$. With these substitutions Eq. 2.36 reduces to:

$$\tan \theta_1 = \frac{r_1 \pi + (a_1 - r_1)\, \theta_1}{a_1 - r_1} \tag{2.40}$$

(This case can also be reduced to the condition of a symmetrical restoring force by letting $a_1 = r_1$; tan θ_1 goes to infinity and θ_1 goes to $\pi/2$, the condition of symmetry as mentioned previously.) When $r_1 >> a_1$, Eq. 2.40 becomes:

$$\tan \theta_1 \cong \theta_1 - \pi \tag{2.41}$$

and Eq. 2.41 is satisfied when $\theta_1 = \pi$ which is the condition of extreme asymmetry with motion only in the positive x-direction. For $a_1 >> r_1$ Eq. 2.40 becomes:

$$\tan \theta_1 \cong \theta_1 \tag{2.42}$$

which is satisfied for $\theta_1 = 0$. Again this is a highly asymmetrical case, but the motion of the moored vessel is now only in the negative x-direction.

2.1.3. Linear Symmetrical Spring System with Free Travel

An interesting restoring force system which bears some resemblance to the physical problem under consideration is the case of a linear symmetrical spring system with free travel, such as shown in Fig. 2.2c. This system is interesting because the restoring force becomes non-linear and the exact solution to the free oscillations of a mass restrained in this way has been reported in the literature (Den Hartog (1956)); therefore, it provides results which can be compared readily to the method developed in Section 2.1.2. It also demonstrates how a simple approximation to the restoring force function can lead to erroneous results.

The example which is chosen is hypothetical and consists of a mass of 260 slugs (lbs sec^2/ft) restrained by linear springs which allow a free travel of $\Delta_f = 0.8$ feet before the restoring force is introduced. The expression which describes this restoring force is given as:

$$F_r = (1.378 \times 10^4)(x - \Delta_f) \tag{2.43}$$

This expression is presented in Fig. 2.5 where the abscissa is the applied force F_r and the ordinate is the resulting displacement. An approximation that is usually made to such restoring force systems is that it is adequately described by an expression of the form $F_r = Kx^n$. It is seen in Fig. 2.5 that, for this case, approximation to the restoring force is poor. Nevertheless a curve of this form is fitted to Eq. 2.43 and for a best fit to the given curve over the region of large forces the following expression is obtained:

$$F_r = 1740\, x^{6.3} \tag{2.44}$$

Den Hartog (1956) has derived the following expression for the period of the free oscillations of a mass restrained by linear springs with free travel:

$$\tau = 2\pi \sqrt{\frac{M}{K}} \left(1 + \frac{2}{\pi} \frac{\Delta_f}{X - \Delta_f}\right) \tag{2.45}$$

where M is the mass, K is the spring constant of the springs used, and Δ_f is the amount of free travel. Eq. 2.45 shows that indeed the dynamic response of this mass is non-linear, i.e., the period τ varies with the amount of initial deflection X.

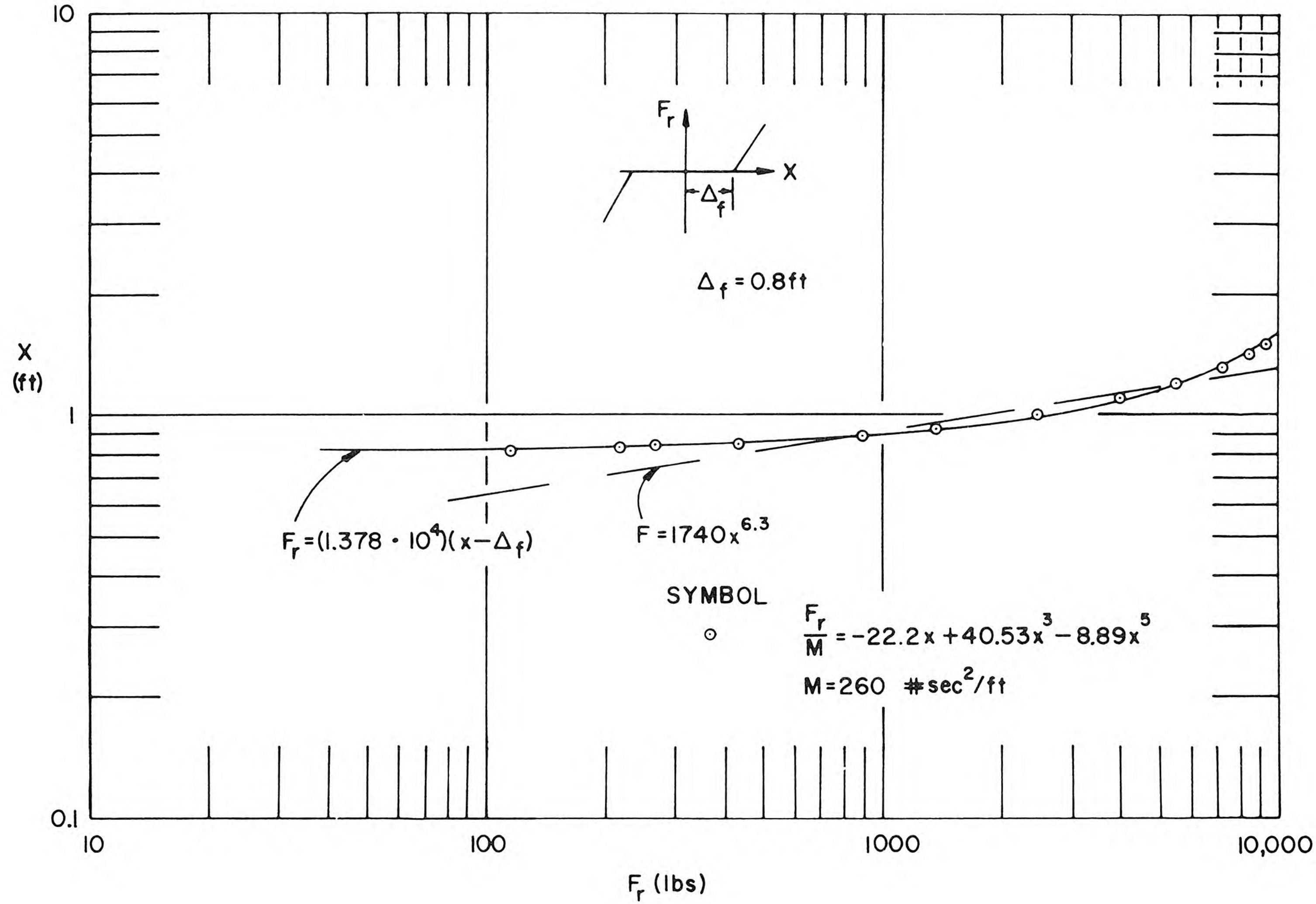

Fig. 2.5. Restoring Force vs. Displacement for Linear Springs with Free Travel (after Fig. 2.2c).

In order to apply the method described in Section 2.1.2 to determine the free oscillations, a polynomial of the form of Eq. 2.33a is fitted to Eq. 2.43; the result is presented in Fig. 2.5. The periods of the free oscillations obtained from Eq. 2.45 are presented in Fig. 2.6. This shows that for deflections less than Δ_f, the free travel, there is no defined period of oscillation since there is no restoring force. As the deflection increases the period of oscillation decreases. The free oscillations obtained from Eq. 2.38 are also shown in Fig. 2.6 and the values agree quite well with Eq. 2.45. A third curve is shown in Fig. 2.6 which was derived in accordance with the approach of Wilson (1958) using Eq. 2.44 to represent the restoring force. It is seen that in general the agreement between this result and the others is poor. This is because Eq. 2.44 does not provide as good an approximation to the restraining force as does the power series of Eq. 2.33. Therefore, in general, for cases where bodies are restrained by a system with significant free travel, it is considered unrealistic to fit an expression of the form $F_r = Kx^n$ to the restoring-force curve and expect to obtain the dynamic response of the body within a reasonable degree of accuracy.

2.2 Restoring Force for Moored Small Boats

In the previous sections the discussion has dealt primarily with the development of the dynamic response of a moored body given the restoring force which restrains the motion of the body in surge. In this section the variation of the restoring force with boat displacement will be determined as a function of the elastic characteristics of the mooring lines and the geometry of the mooring system.

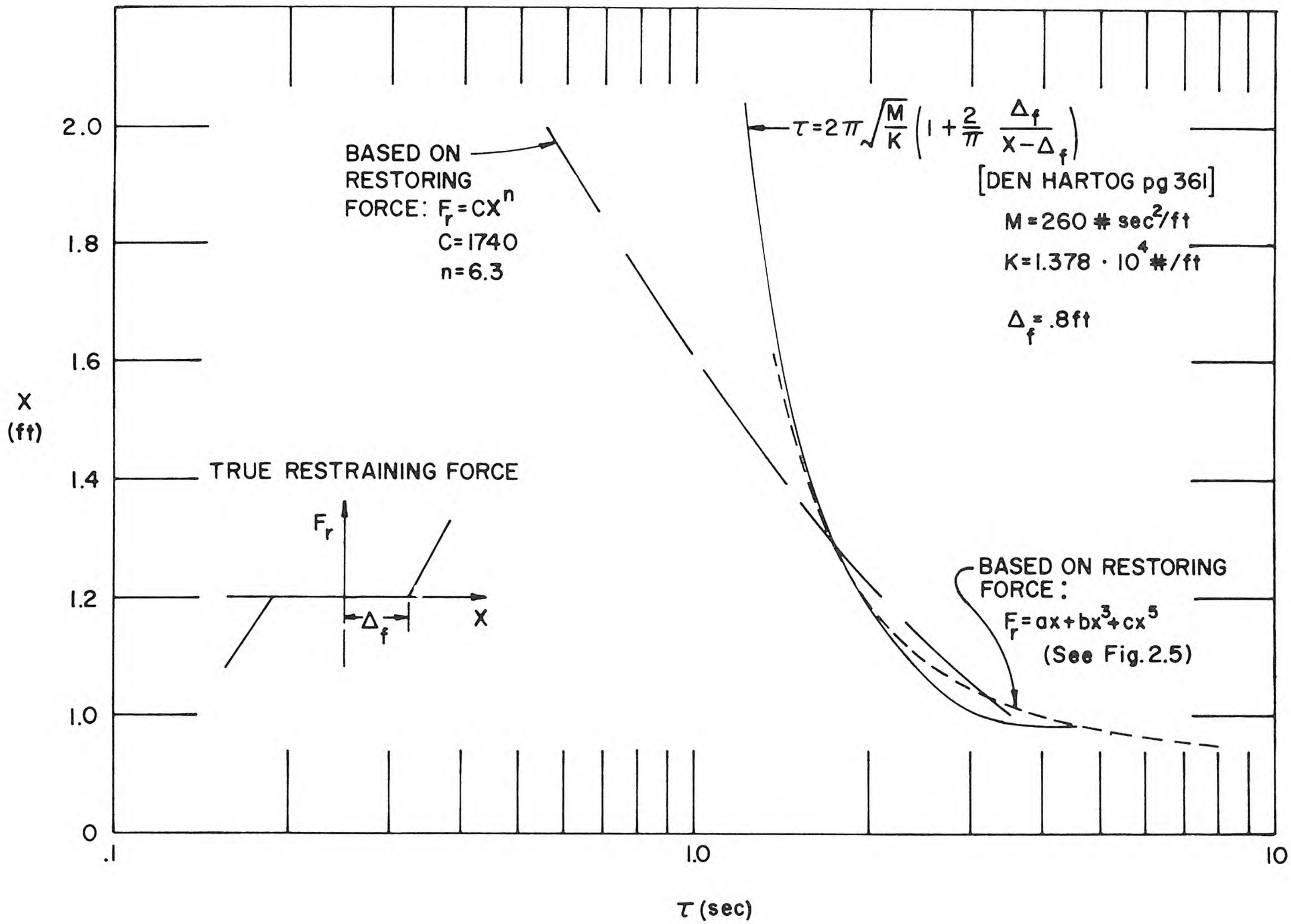

Fig. 2.6. Free Oscillations of Mass Restrained by Linear Springs with Free Travel

Consider first the schematic diagram of Fig. 2.7 which shows a block body (representing the small boat) moored with four lines to a dock. Initially the lines are slack by some arbitrary mount $\Delta \ell$. Upon movement of the body in the positive x-direction this slack becomes zero after the boat moves through a distance Δ_f denoted as the free travel. Until the vessel has moved this distance the restoring force is considered to be zero. This is peculiar to the small boat case where the lines used usually have a small unit weight and the restoring force is considered to develop only due to the elastic characteristics of the lines. Therefore, when $x = \Delta_f$ the line tension is equal to zero ($T^* = 0$), and T^* becomes greater than zero for $x > \Delta_f$.

The restoring force F_r shown in the plan view of Fig. 2.7 is equal to the sum of the x-components of the line tensions, T^*, for all lines acting to restrain the boat's motion in a given direction. Hence:

$$F_r = \sum_{n=1}^{N} T^*_{x_n} \tag{2.46}$$

where $T^*_{x_n}$ is the x-component of the tension in one line and N is the total number of lines restraining the boat from motion in a given direction. From the geometry shown in Fig. 2.7:

$$T^*_{x_n} = T^*_n \cos \beta_n \cos \alpha_n \tag{2.47}$$

It should be noted that the implicit assumption in Eqs. 2.46 and 2.47 is that only boat motion in surge is being considered, i.e., pitch, yaw, and roll are considered negligible. This may or may not be true; however, this assumption is consistent with the previous development

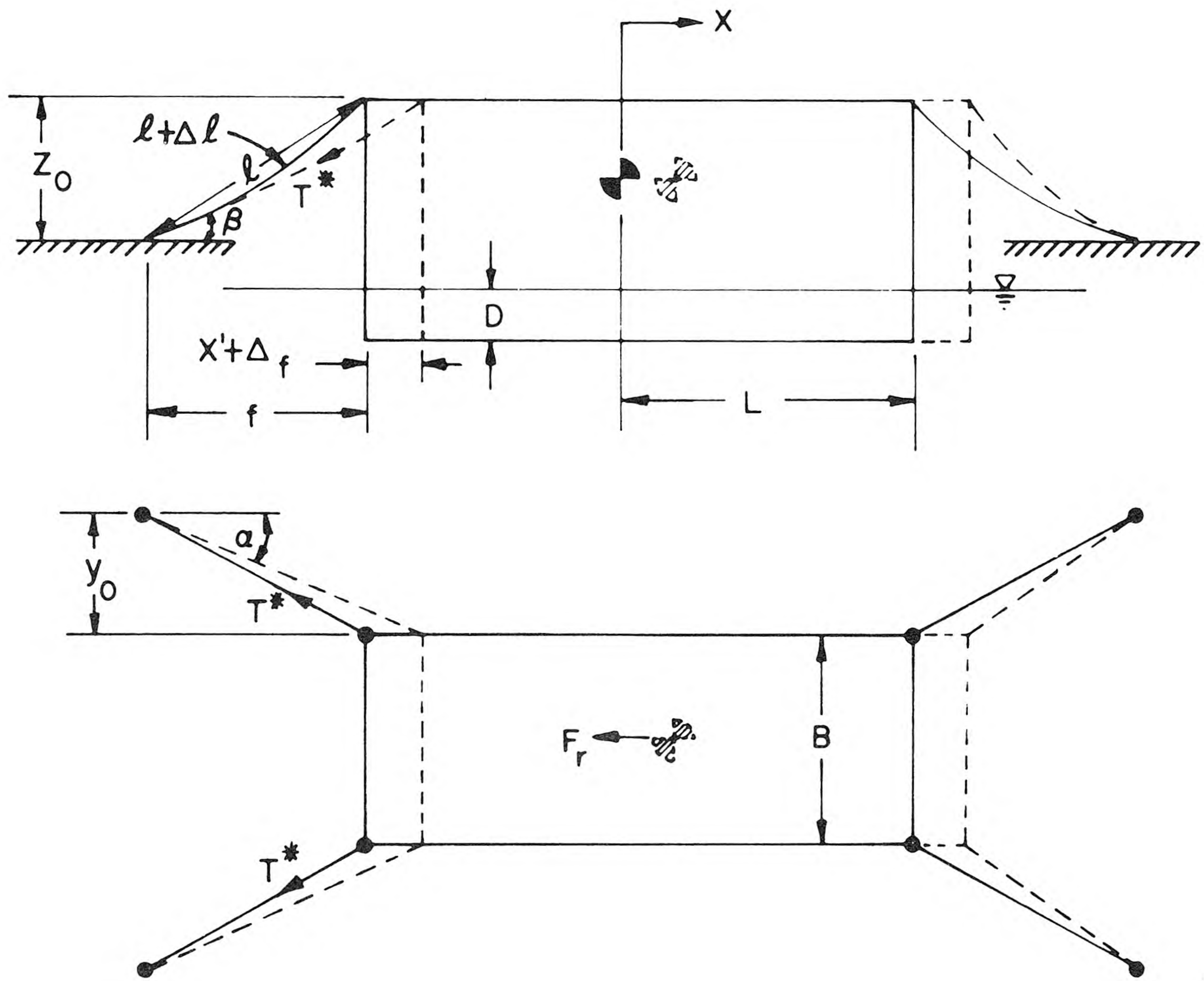

Fig. 2.7. Schematic Diagram of Four-Point Mooring System

and it is felt that the results which this approach yields will describe some of the salient features of the motions of small moored boats. In fact since there is a vertical component of restoring force arising from the line tension the boat will pitch as it moves in surge an amount such that the change in buoyancy equals the change in this component. This does not appear to have a significant effect on the restoring force or the periods of oscillation as will be shown in Section 4.

It is assumed that the elastic characteristics of the mooring line can be represented as:

$$\frac{T^*}{T^*_{Brk.}} = R\,\epsilon^m \tag{2.48}$$

where $T^*_{Brk.}$ is the average breaking strength of the particular line, ϵ is the strain (total elongation divided by length), and R and m are constants. It will be shown in Section 3 that Eq. 2.48 is a reasonable approximation to the stress-strain curve for the types of mooring lines and the limited range of elongation which are of interest in this study.

Consider the definition sketch of a typical line shown in Fig. 2.8. In this case it is assumed that the line goes from a cleat on the dock to the boat, passing through a guide at its point of first contact with the boat and finally it is fastened to a cleat or bit on the boat. Therefore, the line is divided into two sections: the first section going from dock to boat with a length $\ell + \Delta\ell$, and the second section on the boat with a length ℓ_1.

It is assumed that for relatively small movements of the boat and reasonably small line slack compared to the length of lines that the

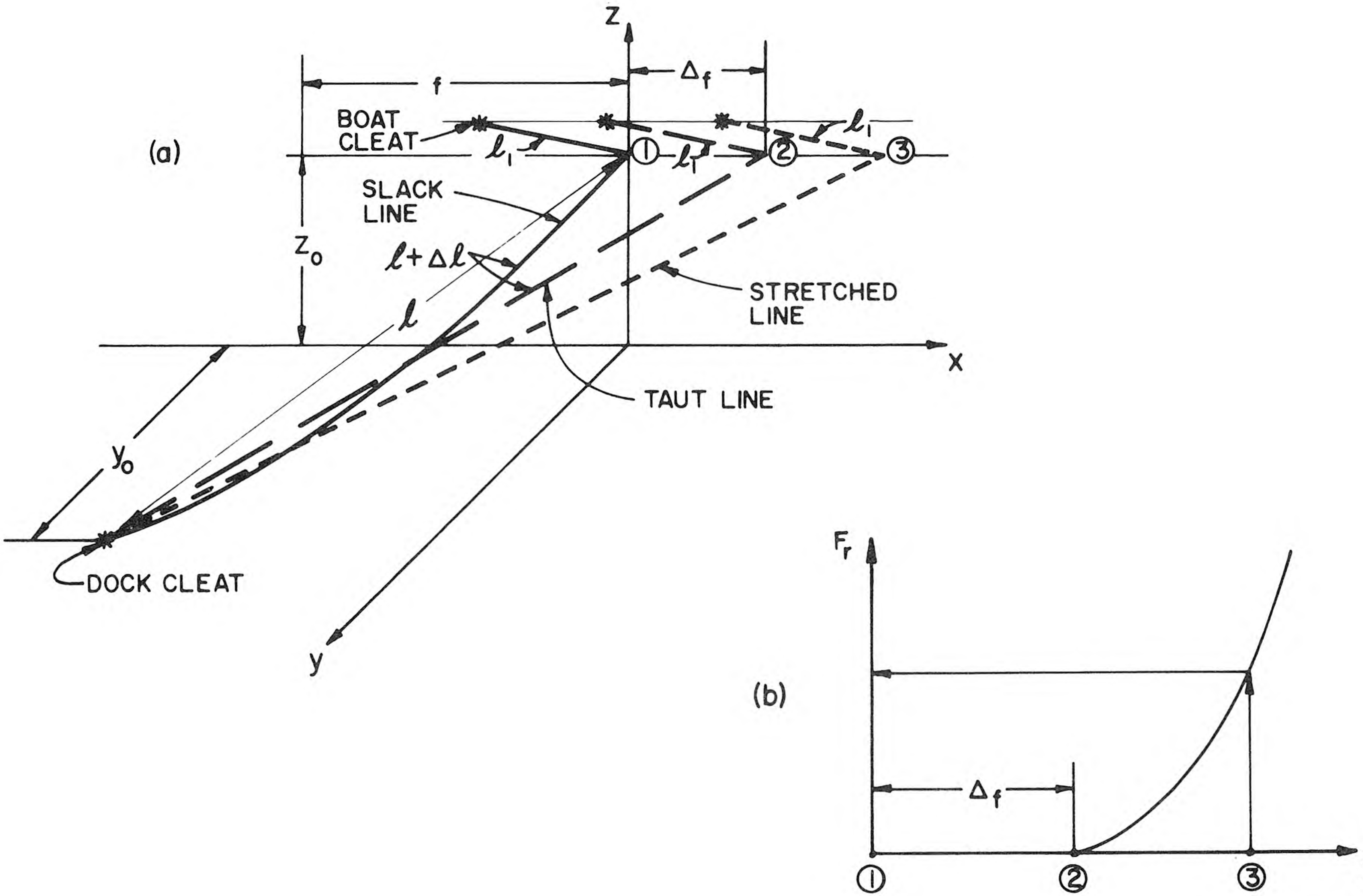

Fig. 2.8. (a) Geometry of Typical Morring Line; (b) Schematic Diagram of Non-Linear Restoring Force with Free Travel

angles α and β do not change significantly after the boat has moved through the distance of free travel Δ_f. Therefore, using the notation of Fig. 2.8 these angles can be expressed as:

$$\cos\alpha = \left[1 - \frac{y_o^{\,2}}{(\ell+\Delta\ell)^2}\right]^{\frac{1}{2}} \tag{2.49a}$$

$$\cos\beta = \left[1 - \frac{z_o^{\,2}}{(\ell+\Delta\ell)^2}\right]^{\frac{1}{2}} \tag{2.49b}$$

The free movement of the boat from point 1 to point 2 in Fig. 2.8 can be expressed as:

$$\Delta_f = \left[(\ell+\Delta\ell)^2 - y_o^{\,2} - z_o^{\,2}\right]^{\frac{1}{2}} - f \tag{2.50}$$

and since $\ell^2 = f^2 + y_o^{\,2} + z_o^{\,2}$, Eq. 2.50 can be rewritten as:

$$\Delta_f = \left[f^2 + 2\,\Delta\ell(\ell) + (\Delta\ell)^2\right]^{\frac{1}{2}} - f \tag{2.51}$$

If the movement from point 2 to point 3 is defined as x', i.e., $x' = x - \Delta_f$, then this displacement can be determined from the applied force and the elastic characteristics of the lines (from Eq. 2.48) once the distance x' is defined in terms of geometry.

$$x' = \left\{\left[(\ell' + \epsilon\ell'')\cos\beta\right]^2 - y_o^{\,2}\right\}^{\frac{1}{2}} - (f + \Delta_f) \tag{2.52}$$

where: $\ell' = \ell + \Delta\ell$

$\ell'' = \ell' + \ell_1$

Eq. 2.52 can be rewritten using Eq. 2.49b as:

$$x' + (f + \Delta_f) = \left\{(\ell')^2 \left[1 + \frac{2\epsilon\ell''}{\ell'} + \left(\frac{\epsilon\ell''}{\ell'}\right)^2\right]\left[1 - \left(\frac{z_o}{\ell'}\right)^2\right] - y_o^2\right\}^{\frac{1}{2}}$$

or: $$x' + (f + \Delta_f) = \ell' \left[\left(\frac{f'}{\ell'}\right)^2 + A\right]^{\frac{1}{2}} \tag{2.53}$$

where:

$$A = \left[1 - \left(\frac{z_o}{\ell'}\right)^2\right]\left[\frac{2\epsilon\ell''}{\ell'} + \left(\frac{\epsilon\ell''}{\ell'}\right)^2\right]$$

$$f' = f + \Delta_f$$

$$(f')^2 = (\ell')^2 - y_o^2 - z_o^2$$

Eq. 2.53 can be expanded as:

$$x' + (f+\Delta_f) = \ell' \left\{\frac{f'}{\ell'} + \frac{1}{2}\left(\frac{f'}{\ell'}\right)^{-1} A - \frac{1}{8}\left(\frac{f'}{\ell'}\right)^{-3} A^2 + \ldots\right\} \tag{2.54}$$

For small elongations of the line and $\epsilon\ell'' << \ell'$ the quantity A becomes:

$$A \cong 2\,\frac{\epsilon\ell''}{\ell'}\left\{1 - \left[\frac{z_o}{\ell'}\right]^2\right\} \tag{2.55}$$

Since the ratio z_o/ℓ' is usually less than unity (but not much less than unity), terms of order A^2 and higher are neglected in Eq. 2.54. (The third term and other higher order terms, in the series of Eq. 2.54 can be shown to be of order ϵ and smaller when compared to the second term and hence justifiably neglected.) Therefore, with these simplifications the following expression is obtained for the boat displacement x' due to the elastic elongation of the line:

$$x' \cong \frac{\ell'}{f'}\left\{1 - \left[\frac{z_o}{\ell'}\right]^2\right\}\epsilon\ell'' \tag{2.56a}$$

or the line elongation is:

$$\epsilon\ell'' \cong \frac{x - \Delta_f}{\frac{\ell'}{f'}\left\{1 - \left[\frac{z_o}{\ell'}\right]^2\right\}} \tag{2.56b}$$

Therefore,the tension in the line can be determined from Eqs. 2.48 and 2.56b as:

$$T^* \cong K_1 \left[\frac{x - \Delta_f}{\ell''\left(\frac{\ell'}{f'}\right)\left[1 - \left(\frac{z_o}{\ell'}\right)^2\right]}\right]^m \tag{2.57}$$

where $K_1 = R\, T^*_{Brk.}$. Substituting Eq. 2.57 into Eq. 2.47 the following expression is obtained for the component of tension in the x-direction of a particular mooring line in terms of its elastic characteristics and its mooring geometry:

$$T^*_{x_n} \cong K_1 \left[\frac{f + \Delta_f}{\ell'(\ell' + \ell_1)}\right]^m \frac{\left[1 - \left(\frac{y_o}{\ell'}\right)^2\right]^{\frac{1}{2}}}{\left[1 - \left(\frac{z_o}{\ell'}\right)^2\right]^{(m - \frac{1}{2})}} \left[x - \Delta_f\right]^m \tag{2.58}$$

The method of approach used in determining the variation of the restoring force with the displacement of the boat in the direction of either the bow or the stern follows directly from Eqs. 2.46 and 2.58. For a given boat displacement, x, the components of the line tensions, $T^*_{x_n}$, computed from Eq. 2.58 and the corresponding elastic characteristics K_1 and m, are summed in accordance with Eq. 2.46. Repeating this for a number of arbitrary displacements, x, which are greater than Δ_f the resultant restoring force curve is obtained as F_r as a function of x (see the schematic curve in Fig. 2.8).

3. ELASTIC CHARACTERISTICS OF MOORING LINES

In order to determine the dynamic response of a moored boat, as described in Section 2, it is first necessary to predict the variation of the restoring force with boat displacement due to a particular mooring system. Eq. 2.58 shows that once the mooring geometry is known, within the assumptions made, the restoring force can be evaluated if the necessary coefficients which describe the elastic nature of the lines (K_1 and m) are known. Two sources of information have been used to evaluate these elastic characteristics in this study: a series of laboratory tests and information supplied by manufacturers.

3.1 Laboratory Tests

A typical section of a mooring line was tested in the laboratory to determine its tensile properties. The rope used was a length of a three-strand twisted-standard lay manila rope with a 5/8-inch nominal diameter. (A sample of the rope is shown in Fig. 3.1 splayed to show the strands.) This test specimen was new and it was taken from a line which was used in a prototype study of mooring systems which will be discussed in detail in Section 4. The rope used in the tension tests is also shown in Fig. 3.1 and it had an initial length of 17.06 inches measured between the attached steel mounting blocks. These blocks were attached to the rope to facilitate testing and are fixed to the rope by means of a commercial polyester resin (Polyester 4130 American Cyanimid Boat Resin). Each steel block was machined with a tapered hole running its full length. The rope was then fitted through the small end of the hole, splayed and set in the resin. In this way the rope was bonded to the mounting blocks in such a way that as a tensile

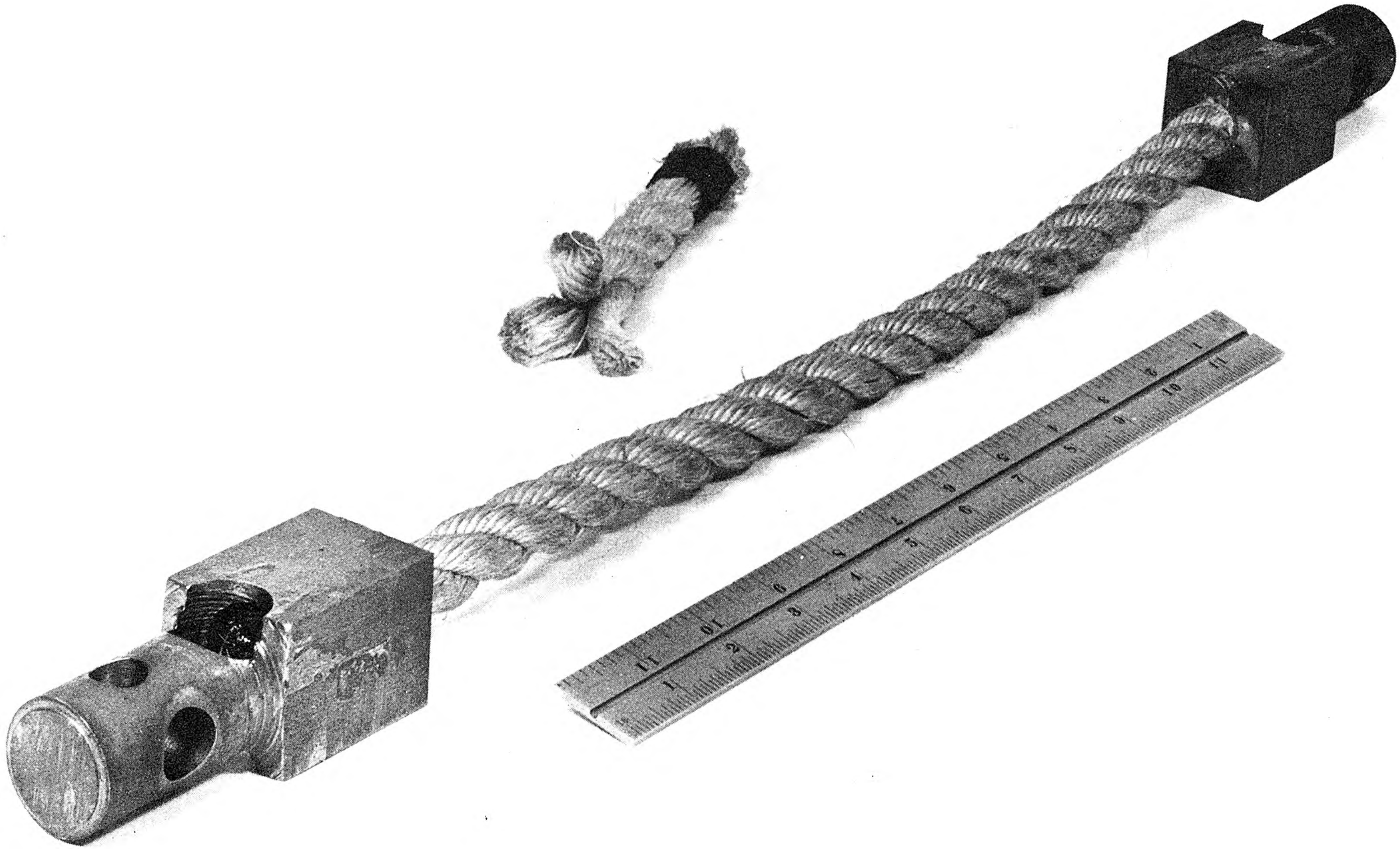

Fig. 3.1. Sample of 5/8-inch Nominal Diameter Manila Rope and Test Specimen

force was applied, the cone-shaped resin would form an even stronger bond with the blocks. It was observed that the resin formed a perfectly adequate bond with the steel over the range of applied loads and there was no observable slip between the rope and the steel mounting blocks.

The specimen was mounted in an Instron Tensile Testing Instrument (MODEL TTCL) which was used to determine the stress-strain characteristics of the rope. The machine is an automatic instrument that moves the specimen clamping blocks apart at a constant predetermined rate and measures the induced force by means of a calibrated load cell. The electrical output from the load cell drives the stylus of an x-y plotter whose paper speed is proportional to the rate of elongation of the specimen. An advantage of this machine, in addition to the features described, is the ability to easily vary the applied load. For instance, a specimen can be stressed a desired amount and then the direction of motion of the clamping blocks reversed so that the tensile force is reduced to zero. This can be repeated thereby investigating the effect of cyclic loading on the elastic properties of the rope.

The results of these tests are presented in Fig. 3.2 which shows the variation of the strain (per cent elongation) with the resultant tensile force. The breaking strength of this rope, obtained from other sources, is approximately 4400 lbs. The different symbols used in this figure denote the results for the indicated loading cycle. The specimen was stressed through a total of 14 cycles. It is interesting,

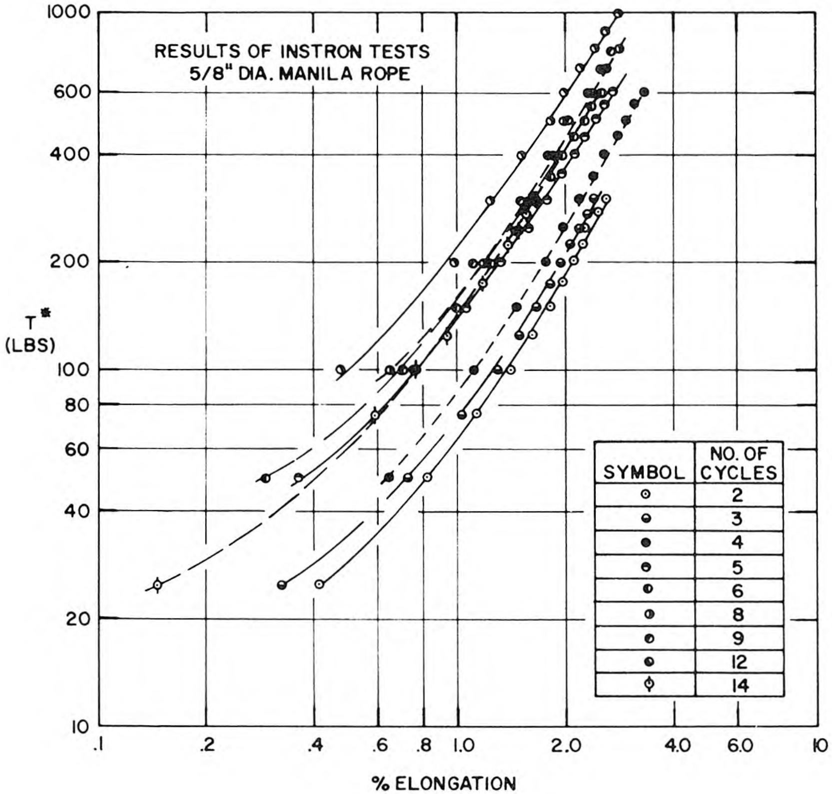

Fig. 3.2. Results of C.I.T. Tension Tests on Manila Rope: Applied Force vs. Percent Elongation

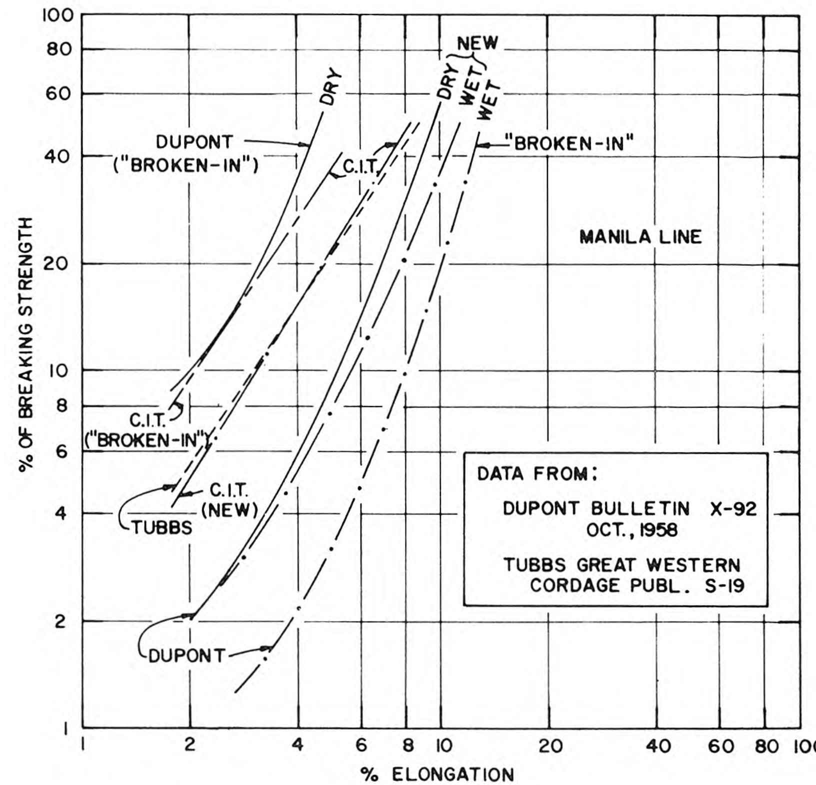

Fig. 3.3. Percent Breaking Strength vs. Percent Elongation: Manila Rope

in the data of Fig. 3.2, that the curves which correspond to 4 cycles of loading and less are grouped together and those for greater than 4 cycles of loading are grouped together. Since the elongation data are referred to the original length (17.06 inches) the latter trend indicates a permanent set as the specimen is repeatedly stressed or "broken-in". In both cases the load-elongation data presented are only for the case of the load being applied; when the load is being removed the results tend to follow a different relation. These data show that for this rope, over the range tested, the tensile force can be expressed approximately in the form:

$$T^* \sim \epsilon^m \qquad (3.1)$$

where ϵ is the elongation. This same form for the variation of tensile force with elongation is proposed by Wilson (1967b).

3.2 Manufacturers' Data

Additional information on manila rope has been obtained from two manufacturers (Tubbs Great Western Cordage (1967) and E. I. DuPont de Nemours and Co., Inc. (1958)). These data are presented in Fig. 3.3 for manila rope as the variation of the tensile force in per cent of the breaking strength with the percent elongation for different loading conditions. In addition, average data from the laboratory tests shown in Fig. 3.2 (indicated in Fig. 3.3 as C.I.T.) are included. The breaking strength of the ropes used was 4400 lbs. Referring to Fig. 3.3 it is seen that the tests run by DuPont were for two different rope conditions: new rope and "broken-in" rope, where the latter refers to the rope after repeated loadings. In addition to these conditions tests were run with the specimen dry and wet. These data indicate, for the new rope, approximately the same stress-strain relationship independent of whether the rope is wet or dry. However, the relations are radically different for the rope after repeated loadings. Only one curve is reported by Tubbs Great Western Cordage and this is referred to in their publication as the approximate working elasticity after permanent elongation; hence, this should correspond more to the case of "broken-in" lines.

From these data, at best one can say that for manila rope one would expect a relatively wide variation in restoring force acting on a moored boat depending on the condition of the rope. Therefore, it would be difficult to predict exactly the dynamic response of the moored boat from the type of line and the boat-mooring geometry. However, it is felt that a reasonably reliable estimate can be made from the judicious use of data such as that presented in Fig. 3.3. This will be discussed more fully in Section 4.

Information similar to that obtained for manila rope has been obtained for Nylon rope from the manufacturers. These data are presented in Fig. 3.4 as the variation of the applied load expressed as a percentage of the breaking strength with percent elongation. As seen in Fig. 3.4 the data from the manufacturers is not nearly as variable for Nylon as for manila. An interesting difference between the two materials is that although Nylon becomes stiffer as it is "broken-in", again probably due to an adjustment of the fibers, the shift is not as great as for manila. The information from Tubbs Great Western

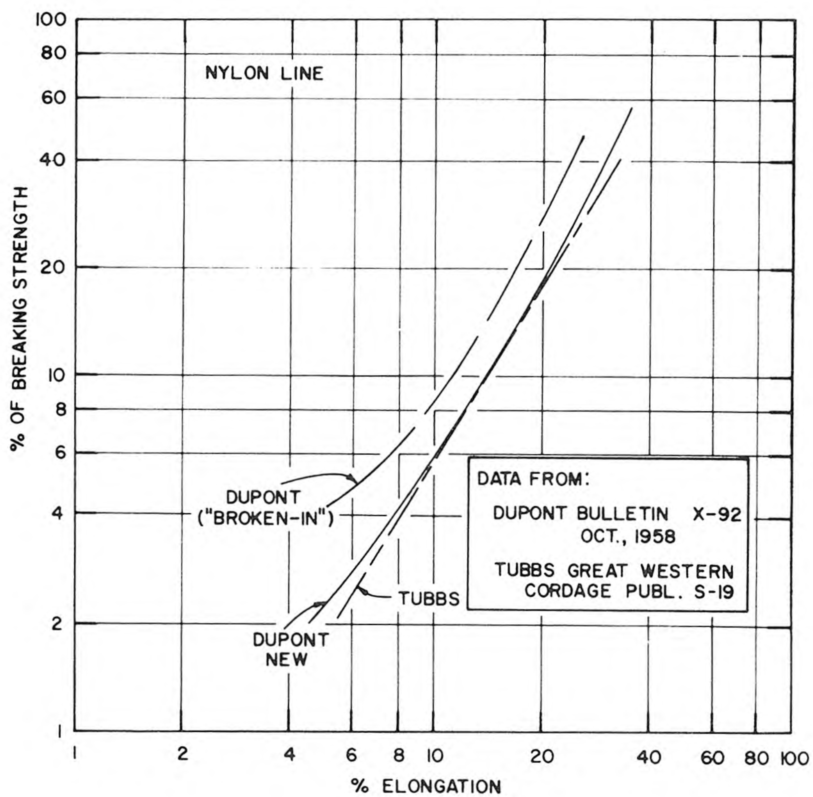

Fig. 3.4. Percent Breaking Strength vs. Percent Elongation: Nylon Rope

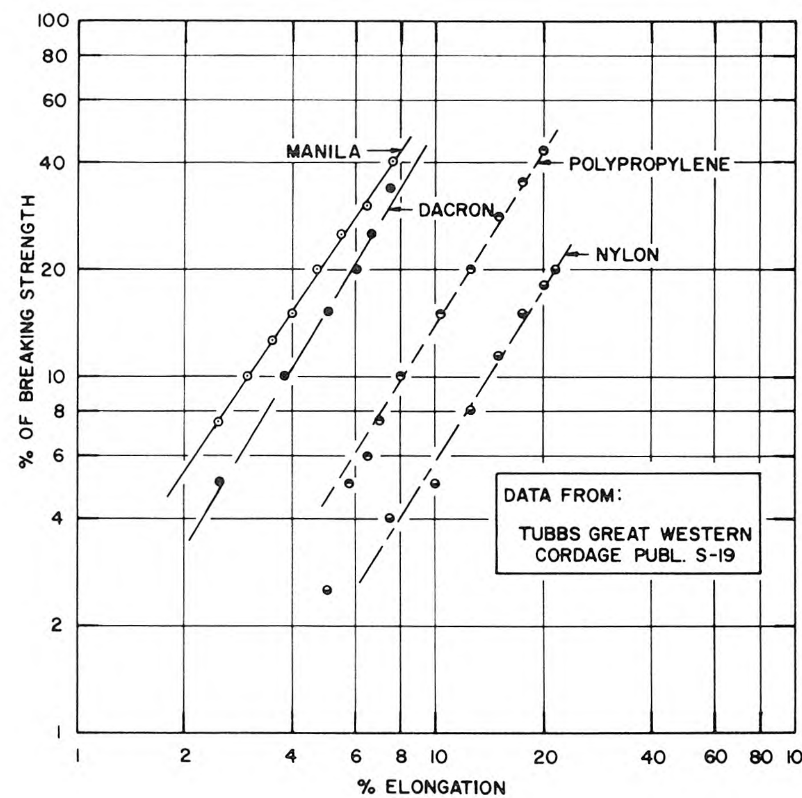

Fig. 3.5. Percent Breaking Strength vs. Percent Elongation: Various Ropes

Cordage appears to agree better with the DuPont results for new rope than for "broken-in" line. For both series of tests Nylon appears to be a material which is more elastic than manila and would result in larger boat motions for the same applied force in the absence of dynamic effects. For Nylon ropes an expression of the form of Eq. 3.1 also appears to fit these data reasonably well.

In Fig. 3.5 stress-strain curves are presented for four different materials from data obtained from Tubbs Great Western Cordage. The data for manila and Nylon have been presented previously in Figs. 3.3 and 3.4. It is seen that all data appear to be of the form described by Eq. 3.1. The coefficients shown in Table 3.1 have been obtained from fitting Eq. 2.48 to the data of Fig. 3.5.

Table 3.1. Coefficients in Stress-Strain Relation:

$$\frac{T^*}{T^*_{Brk}} = R\,\epsilon^m$$

MATERIAL	R	m
MANILA	18.	1.48
DACRON	25.15	1.70
POLYPROPYLENE	5.72	1.62
NYLON	2.54	1.65

In addition to these data, information on the breaking strength of various ropes is presented in Table 3.2. These data were extracted from Tubbs Great Western Cordate (1967) and when used in conjunction with information such as that presented in Table 3.1 one can obtain the constants K_1 and m in Eq. 2.58. (Wilson (1967b) has also presented information on the breaking strength of ropes of various materials which can be used in lieu of the data presented in Table 3.2.) Therefore, from the mooring dimensions and the information from Tables 3.1 and 3.2 the variation of the restoring force with displacement for a particular moored boat can be estimated in accordance with Eq. 2.58. It is felt that although this approach cannot be exact, due to the variation in the elastic characteristics of the lines, it provides data from which a reliable estimate of the boat-mooring dynamics can be obtained.

Table 3.2. Tensile Strength of Various Three-Strand Twisted-Standard Lay Ropes*

SIZES IN INCHES		TENSILE STRENGTH IN POUNDS T^{*}_{BRK}			
		Minimum	Approximate Average		
Circ.	Dia.	Manila	Nylon	Dacron	Polypropylene
5/8	3/16	450	1,000	1,000	800
3/4	1/4	600	1,650	1,650	1,250
1	5/16	1,000	2,550	2,550	1,900
1-1/8	3/8	1,350	3,700	3,700	2,700
1-1/4	7/16	1,750	5,000	5,000	3,500
1-3/8	15/32	2,250	--	--	--
1-1/2	1/2	2,650	6,400	6,400	4,200
1-3/4	9/16	3,450	8,000	8,000	5,100
2	5/8	4,400	10,400	10,000	6,200
2-1/4	3/4	5,400	14,200	12,500	8,500
2-1/2	13/16	6,500	--	--	--
2-3/4	7/8	7,700	20,000	18,000	11,500
3	1	9,000	25,000	22,000	14,000
3-1/4	1- 1/16	10,500	--	--	--
3-1/2	1- 1/8	12,000	33,000	29,500	18,300
3-3/4	1- 1/4	13,500	37,500	33,200	21,000
4	1- 5/16	15,000	43,000	37,500	23,500
4-1/2	1- 1/2	18,500	53,000	46,800	29,700
5	1- 5/8	22,500	65,000	57,000	36,000
5-1/2	1- 3/4	26,500	78,000	67,800	43,000
6	2	31,000	92,000	80,000	52,000
6-1/2	2- 1/8	36,000	106,000	92,000	61,000
7	2- 1/4	41,000	125,000	107,000	69,000
7-1/2	2- 1/2	46,500	140,000	122,000	80,000
8	2- 5/8	52,000	162,000	137,000	90,000
8-1/2	2- 7/8	58,000	--	--	--
9	3	64,000	200,000	174,000	114,000
10	3- 1/4	77,000	250,000	210,000	137,000
11	3- 5/8	91,000	300,000	254,000	162,000
12	4	105,000	360,000	300,000	190,000
RECOMMENDED SAFE WORKING LOAD (Based on the Percent of Tensile Strength)		20%	11%	11%	17%

*Abstracted from Tubbs Great Western Cordage Form 104, Jan. 19, 1967.

4. PROTOTYPE MOORING STUDY: HARBOR BOAT NO. 3

In this section a series of experiments will be discussed which were conducted on a small moored boat. The purpose of the experiments was to study the free oscillations of a small boat moored with bow and stern lines in a typical slip at a local small-boat harbor in order to determine the reliability of the analytical approach presented in Section 2 of this report.

4.1 Description of Boat and Its Mooring System

A photograph of the boat used in these experiments is presented in Fig. 4.1 and it will be referred to in this discussion as Harbor Boat No. 3. The boat was operated by the Harbor Patrol of the Department of Small Craft Harbors, County of Los Angeles, at Marina del Rey (a small-craft harbor located near Los Angeles, California) and was built to their specifications by United Boatbuilders, Inc., Bellingham, Washington. The nominal dimensions of the boat, as transmitted by private communication with the builders, are shown in Table 4.1.

Table 4.1. Nominal Dimensions of Harbor Boat No. 3

Length	26 feet
Beam	9 feet - 2 inches
Maximum Draft	2 feet - 4 inches
Approximate Displacement (unloaded)	5200 lbs

Fig. 4.1. Harbor Boat No. 3; Length = 26 ft, Loaded Displaced Weight $\cong$ 7000 lbs.

From discussions with the harbor personnel it was determined that the displaced weight submitted by the builders was low due to the exclusion of certain special equipment subsequently added after delivery. A more realistic displaced weight was considered to be 7000 lbs. Therefore, in all subsequent calculations the mass of Harbor Boat No. 3 will be based on a displaced weight of 7000 lbs.

Figs. 4.2 and 4.3 show the shape of the boat in some detail. Fig. 4.2 is a drawing of the plan view and elevation of the vessel and Fig. 4.3 is a drawing of the boat-lines. The former figure is self-explanatory; however, some comment is necessary to describe Fig. 4.3.

The shape of hull of the boat is shown in Fig. 4.3 in detail which is sufficient for construction purposes. The upper portion of the figure is the combination of all cross sections of the boat for 2 foot stations from the bow to the stern. One could view this portion of the figure as templates of the hull of the boat. By passing planes through these cross-section shapes parallel to and perpendicular to the water line, the curves drawn in the lower portion of Fig. 4.3 are constructed. This part of the figure shows the outline of the hull which would occur on these horizontal and vertical planes in plan and elevation views. The small draft of this vessel is evident from the nominal water line shown on the drawing (this water line is for the displaced weight presented in Table 4.1). The waterline-length for this displaced weight is approximately 22 feet - 9 inches. Fig. 4.3 shows that the shape of the hull is more of a modified "Vee" shape rather than the block body shape assumed in the analysis. However, for small drafts it is felt that the underwater shape can be reasonably approximated by a block

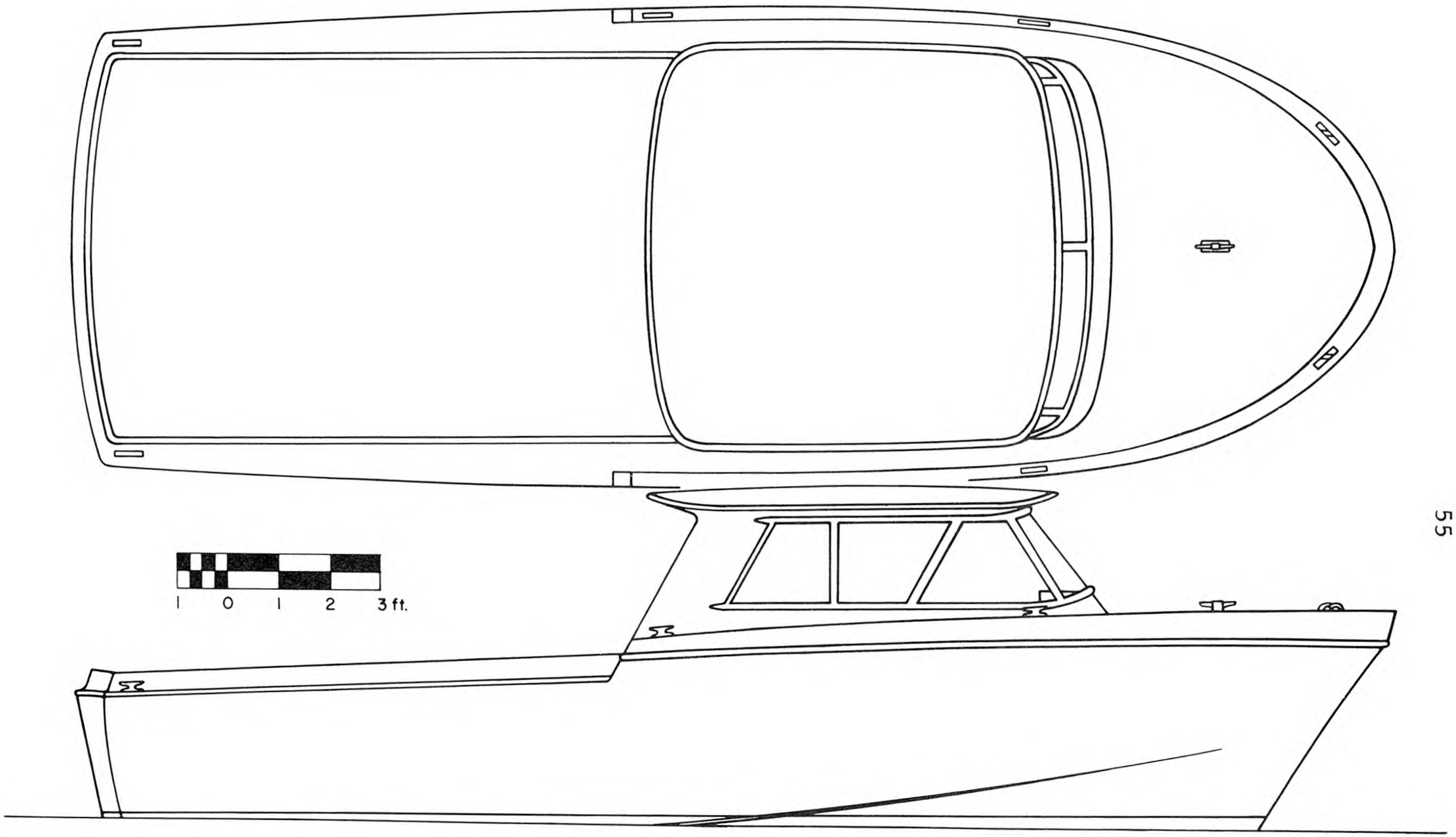

Fig. 4.2. Plan View and Elevation Drawing of Harbor Boat No. 3

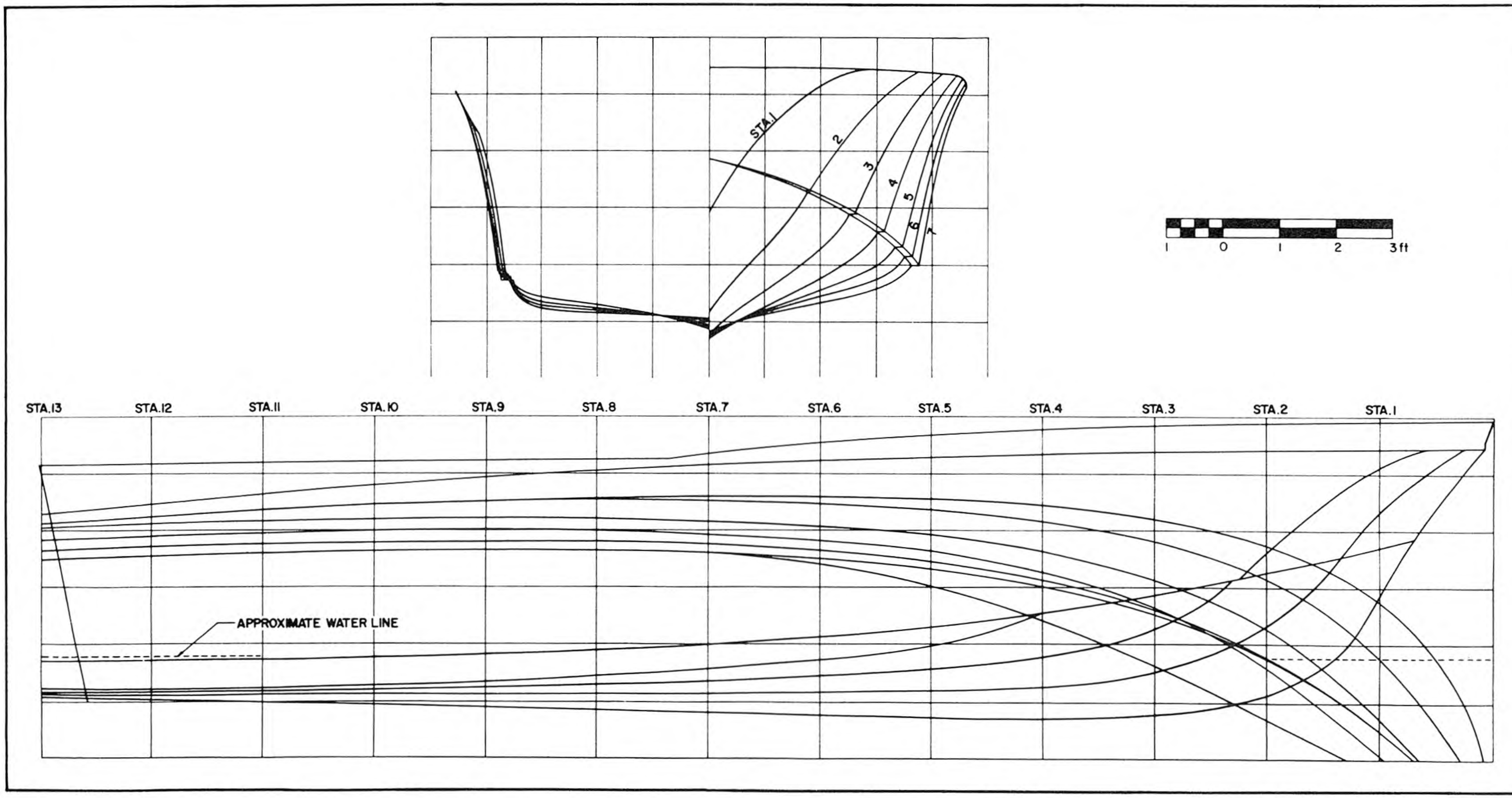

Fig. 4.3. Boat-Lines: Harbor Boat No. 3

body; the length and draft of this approximate shape and the effect of the shape upon the boat response will be discussed fully later.

Various slip configurations are used in small craft harbors and in fact these systems vary in a particular marina. In this study the slip used was floating: free to move vertically but constrained in other directions. The slip is supported by buoyancy chambers located beneath wooden walkways and constrained by piles which essentially act as rails to guide the vertical movement. A portion of one such slip can be seen in the photograph Fig. 1.1. The flotation chambers cannot be seen, but they consist of air-filled rectangular boxes. The slips in general use at Marina del Rey are sized for one boat per slip and a plan view of the slip used for the experimental program is shown in Fig. 4.4. Since the boat has an overall length of approximately 26 feet and a beam of approximately 9 feet there is adequate clearance between the boat and the dock. It should be noted at this point that in general this is not the case, i.e., the clearance between the bow of the boat and the slip is usually much less.

Harbor Boat No. 3 was moored using a four-point mooring system similar to the schematic sketch presented in Fig. 4.4. New 5/8-inch diameter manila rope was used for the mooring and a section of this rope was tested in the laboratory to determine its elastic characteristics (see Fig. 3.2). Using the nomenclature of Fig. 2.8, the dimensions of the mooring lines used are presented in Table 4.2 for the condition of all lines taut.

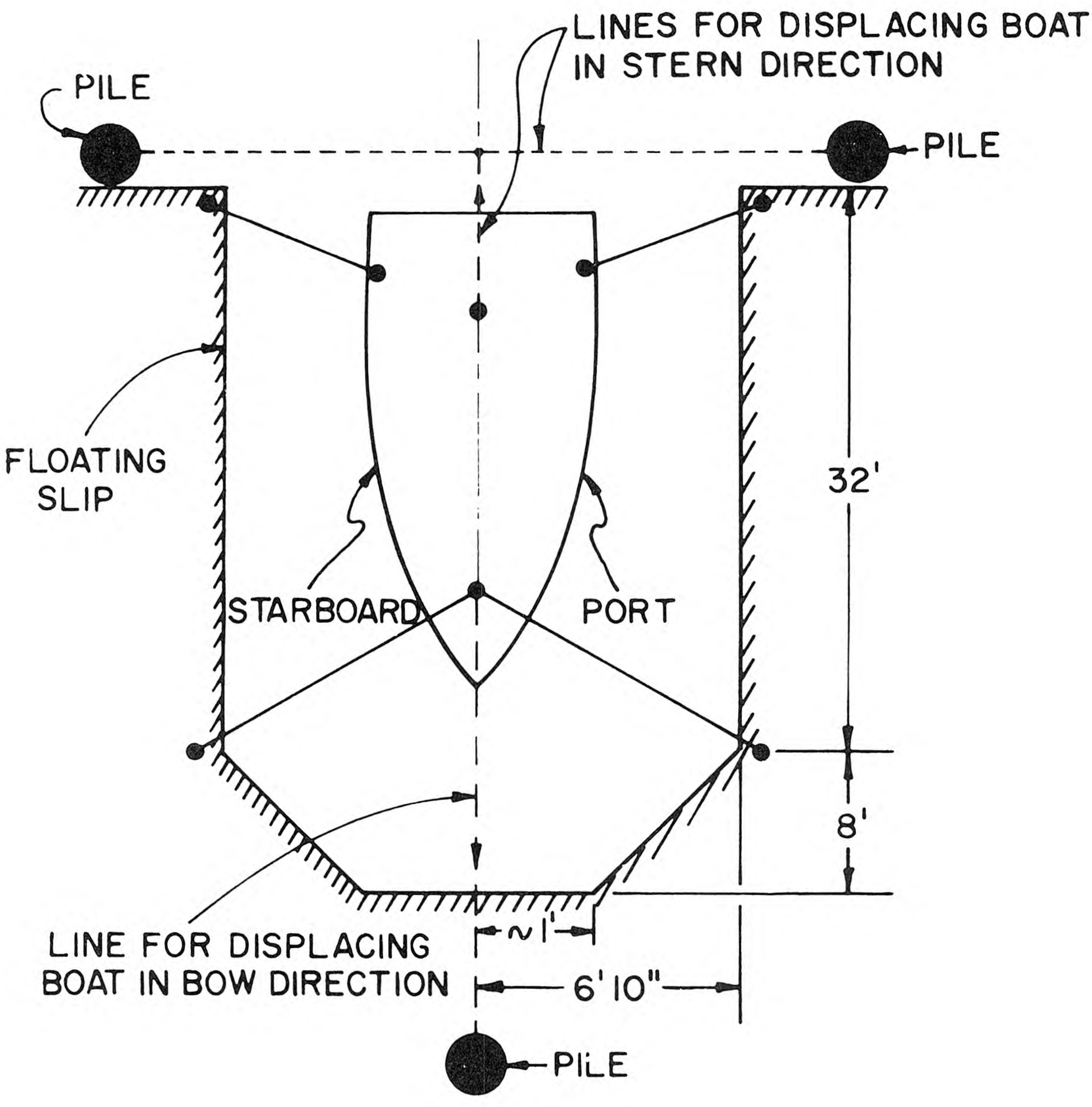

Fig. 4.4. Schematic Drawing of Slip, Mooring System, and Harbor Boat No. 3

Table 4.2. Dimensions of Mooring Lines of Harbor Boat No. 3

	Bow Lines		Stern Lines	
	Port	Starboard	Port	Starboard
z_o	27.5*	29.	18.	16.
y_o	27.	32.	33.	36..
f	60.	57.	76.	80.
ℓ	71.	72.	85.	89.
ℓ_1	36.	36.	0.	0.
	*Note: all dimensions in inches			

It is seen from the data shown in Table 4.2, that although an attempt was made to moor the boat in the center of the slip, this objective was not achieved exactly. The length of mooring line, ℓ, shown in Table 4.2 is for the case of all lines taut; in the tests two other cases were examined: 4 inches and 8 inches of slack in all lines.

4.2 Experimental Equipment and Procedure

The basic principle of the testing was simple, since the ultimate objective of the experiments was to obtain the period of free oscillation of a typical small boat moored in various ways. The procedure was to first displace the boat from the at-rest-position in either the direction of the bow or the stern by a known force and then release it photographing the motion of the boat in surge upon release. A photograph of Harbor Boat No. 3 moored in the test-slip showing some of the experimental equipment used is presented in Fig. 4.5.

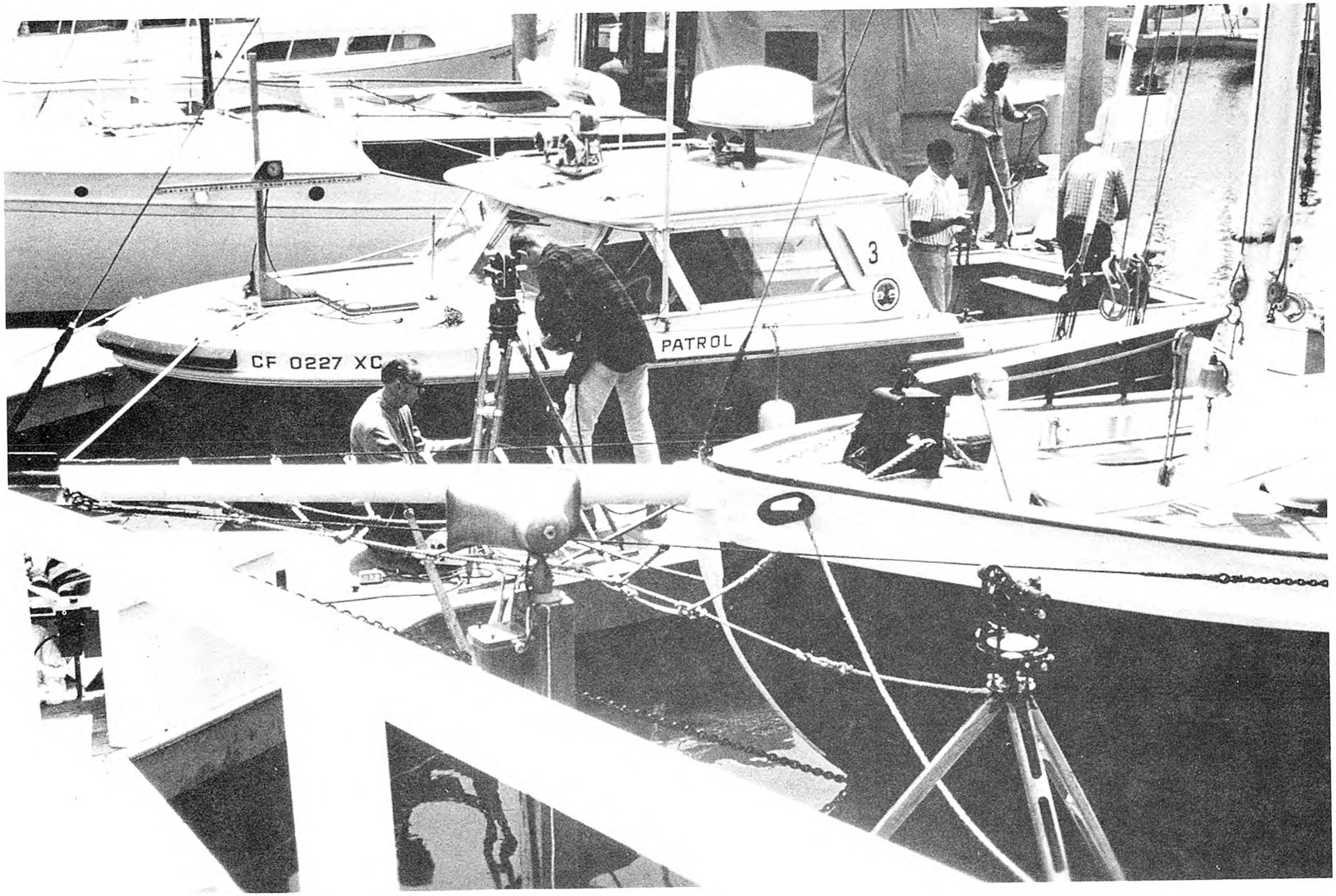

Fig. 4.5. Photograph of Harbor Boat No. 3 and Experimental Equipment

The time-history of boat motion was determined from moving pictures taken of a scale firmly fixed to the boat. The scale can be seen in Fig. 4.5 mounted in a horizontal position to a fixed vertical post on the boat. Also on the boat is an electric stop clock which could be read to within ± 0.05 sec. The movie camera was mounted on a tripod which was located on the floating dock. This camera was a Bolex H16 Reflex and was operated electrically through a specially designed time-lapse control. The time-lapse control permitted the film frames to be advanced in the camera at a predetermined time interval. A photograph of the camera and time-lapse control is shown in Fig. 4.6. The control can be set to advance the film in calibrated intervals of one sec (except for the fastest rate which is at a 0.5 sec interval or at a variable time interval ranging from 0.5 to 2.5 secs. By including the electric clock in the film frame, it was possible to eliminate the need for accurate setting of the time-lapse interval and a time reference could be evaluated directly. In addition to the camera, a surveyor's transit was placed nearby so that a check of the scale movement could be obtained.

By suitable rigging, ropes were attached between the boat and the piles which guided the floating ships. These ropes, as shown in Fig. 4.4, were used to displace the boat in either the positive or negative x-direction. A load cell was mounted between the line and the support pile to determine the force with which the boat was deflected. This load cell was a Martin Decker Tension Load Cell (SD Series) with a scale divided into one pound increments. Between the

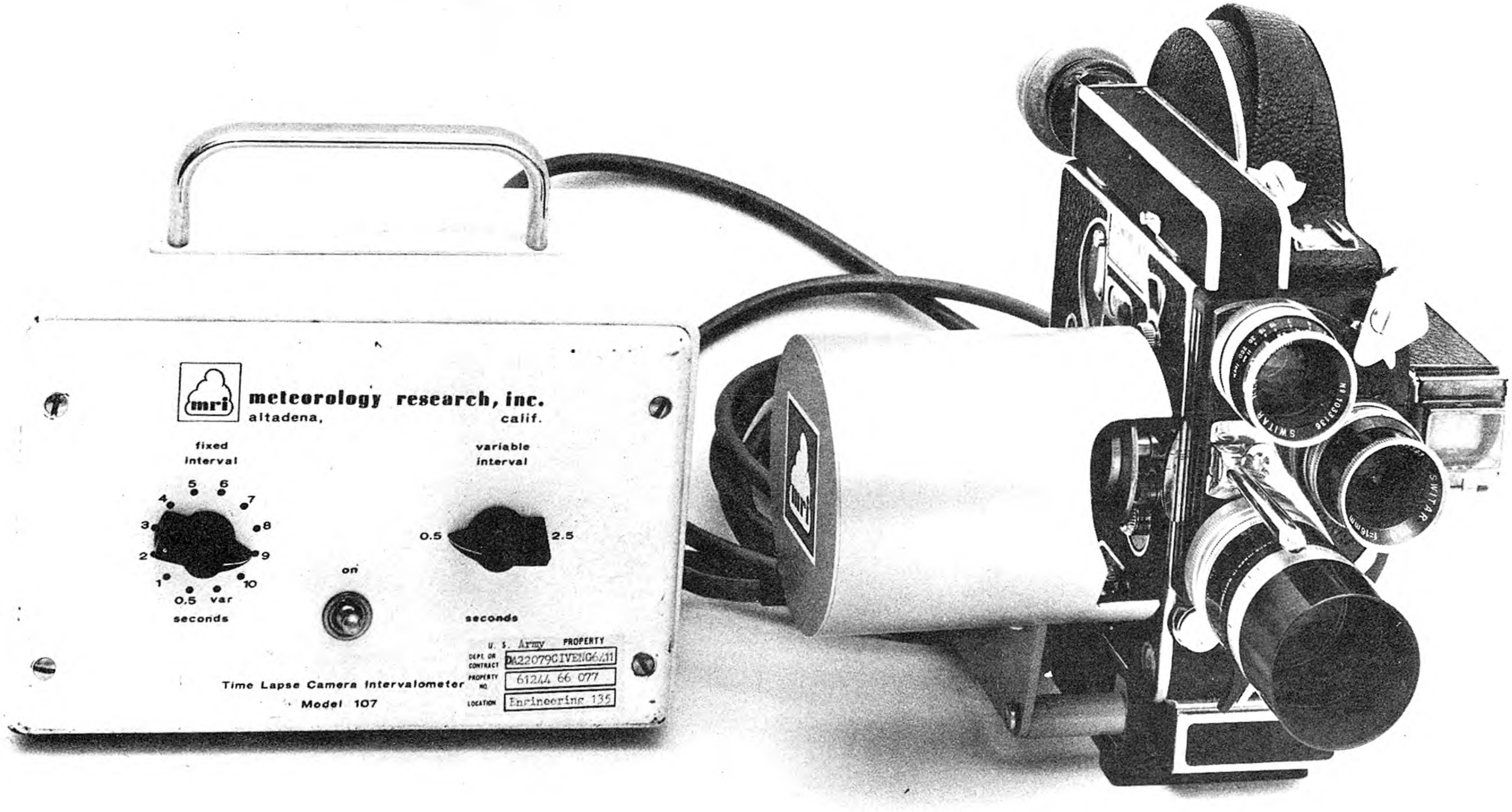

Fig. 4.6. Time Lapse Movie Camera

load cell and the boat the manila line used for displacing the boat had a small loop in it. The upper portion of the loop was held in place by relatively thin nylon cord. When a force was induced in this line resulting in a boat displacement, this force was transmitted from one part of the rope used for displacement through the nylon cord to the other section. The nylon cord was cut after displacing the boat the desired amount thereby suddenly releasing the boat without damaging the "displacement-rope". This had two advantages: the boat could be released nearly instantaneously without the displacement rope interfering with the motion and this rope was not destroyed each time; the former was the most important. A mechanical "chain-fall" was placed between the load-cell and the pile to provide for reasonably large applied forces. However, it was found that the maximum possible force, which could be applied safely without harming the piles, was approximately 500 lbs. Therefore, two series of tests evolved: the first for applied loads of 200 lbs, and the second for applied loads of approximately 500 lbs.

The point of load application on the boat was the forward bit for boat displacements toward the bow from the at-rest-position, and the stern bit for displacements in the direction of the stern. In both cases this point of application was above the center of gravity of the boat, so some pitching was introduced during the initial phases of an experiment. This feature of the tests will be discussed later.

The test procedure was as follows. The boat was displaced by applying measured increments of force determined from the load-cell.

At the same time the boat displacement was recorded on film as well as measured using the transit. This procedure was considered adequate for the case of taut lines. After determining the force-displacement characteristics of one set of mooring lines (bow or stern), the boat was displaced with a predetermined force (either 200 lbs or 500 lbs) and then released by cutting the nylon cord described previously. Since the movie camera had been started prior to the release of the boat a complete time-history of the movement was then recorded on film. These films were analyzed in the laboratory frame-by-frame to determine the displacement-time history during free oscillation. After performing experiments for displacements from the bow, these experiments were repeated for displacements from the stern. Similar experiments were conducted for the cases where line slack was introduced.

4.3 Measured and Predicted Restoring Force

The variation of the restoring force with displacement in surge was determined for three cases: all lines taut, 4 inches slack in all lines, and 8 inches slack in all lines. The results for the case of Harbor Boat No. 3 moored with taut 5/8-inch manila lines are presented in Fig. 4.7 for the mooring system shown schematically in Fig. 4.4 (see also the photograph, Fig. 4.5) and the mooring system dimensions presented in Table 4.2.

In addition to the measured variation of the restoring force with boat displacement, curves which represent the predicted restoring force based on Eqs. 2.46 and 2.58 are shown. The elastic characteristics of the mooring lines used for the predicted restoring forces,

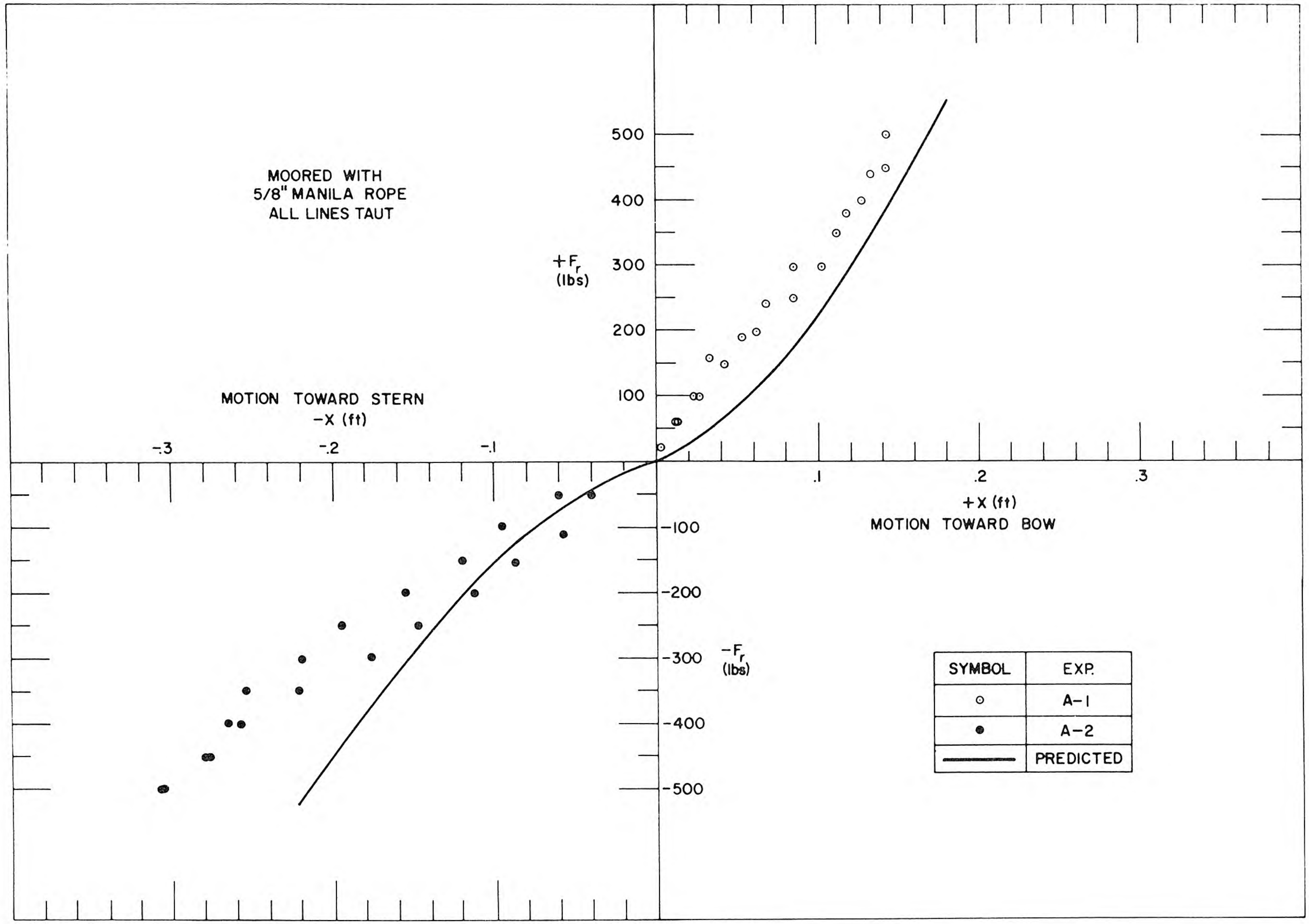

Fig. 4.7. Measured and Predicted Restoring Force vs. Displacement: Harbor Boat No. 3, All Lines Taut

i.e., K_1 and m in Eq. 2.58, were determined from the results of tests of a short length of the rope which was used in the mooring system (see Section 3.1). Referring to Fig. 3.3, the tests which correspond to the curve which is designated as C.I.T. (new) were used to determine these constants. These data were used since they agreed well with those predicted by one of the rope manufacturers (Tubbs Great Western Cordage (1967)), and in addition the lines used in the field for the mooring could not be considered to be well "broken-in" lines. A factor which contributes to this is the difference which may exist between the elastic characteristics of a short section of rope which is exposed to cyclic loading in the laboratory and an actual mooring line five times as long exposed to similar loadings in the field. In the former case, gross readjustments of the fibre orientation may occur over the full length of the test section for a relatively small number of loading cycles; however, in the latter case for a similar loading these adjustments may be more localized, i.e., limited to regions of the line near fittings. Therefore, the elastic characteristics obtained in the laboratory for small loading cycles would probably be the most realistic data to use when predicting the restoring force to be expected for Harbor Boat No. 3.

In Fig. 4.7 the agreement is only relatively good between the actual displacement and the predicted displacement for a given applied force. One major difference between the analytical model and the moored boat is the assumed condition of negligible pitch in the former case. Due to the change in the buoyancy distribution (and, therefore, the location of the center of buoyancy) caused by displacing the boat by a force applied above its center of gravity, the force for a given displacement could be greater or less than that predicted in a model without pitching. Actually the pitch, bow up or stern up, is also a function of the line arrangement so that it is difficult a priori to predict the magnitude and direction of the pitch. The change in the center of buoyancy is correspondingly difficult to predict due to the complex underwater shape of the vessel as shown in Fig. 4.3. Therefore, this simplified model cannot be expected to yield exact information on the expected restoring force for a given displacement.

The measured force-displacement data for Harbor Boat No. 3 moored with all lines with 4 inches and 8 inches slack are presented in Figs. 4.8 and 4.9 respectively. In both cases the results of the tests were somewhat in question due to the difficulty in obtaining an accurate measurement of the at-rest-position of the boat when moored with slack lines. Any small wave action caused the boat to surge to-and-fro and even though this motion was averaged to determine the at-rest-position the result is questionable. For this reason this location was determined, for Figs. 4.8 and 4.9, by taking the total recorded travel in the positive and the negative x-direction when a small load was applied and dividing this result by two. The displacements were then referenced to this redefined zero.

Although the force vs. displacement data obtained experimentally for the cases with line slack are not exact because of these problems, the measurement of the displacement-time history of boat motions during the free oscillations are accurate with respect to the arbitrary at-rest-position. Hence, the periods of free oscillation of the small

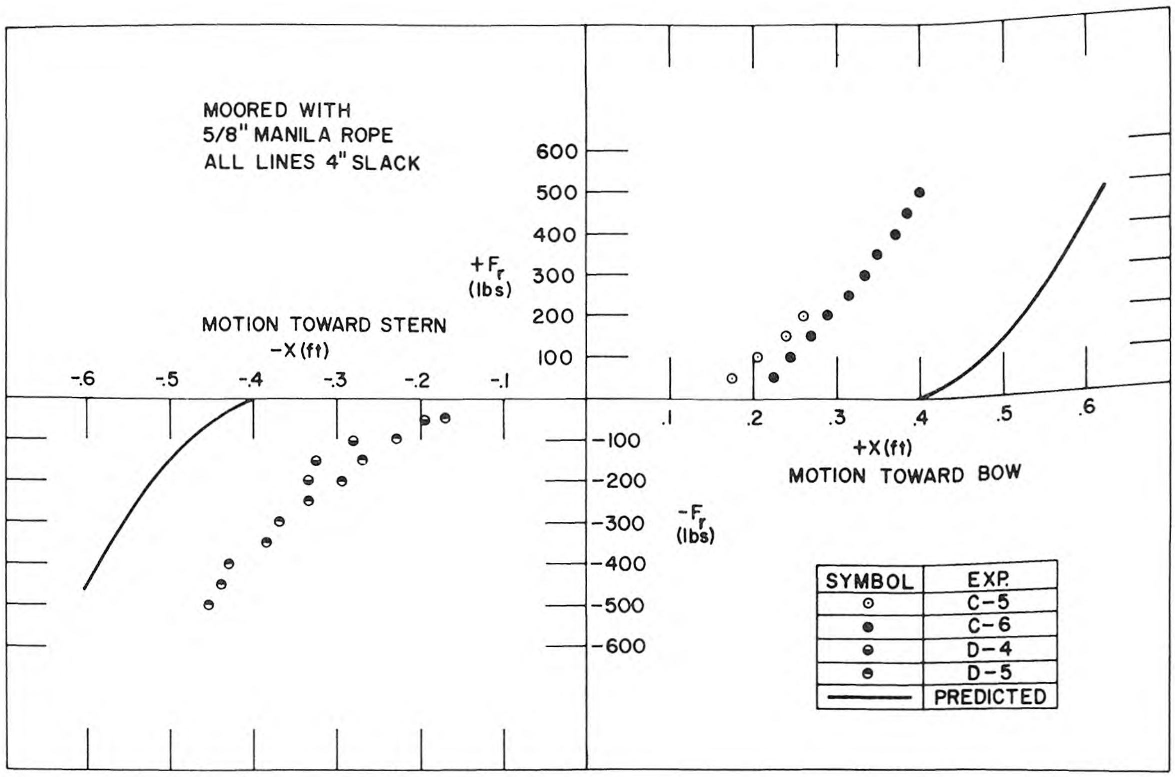

Fig. 4.8. Measured and Predicted Restoring Force vs. Displacement: Harbor Boat No. 3, All Lines 4 Inches Slack

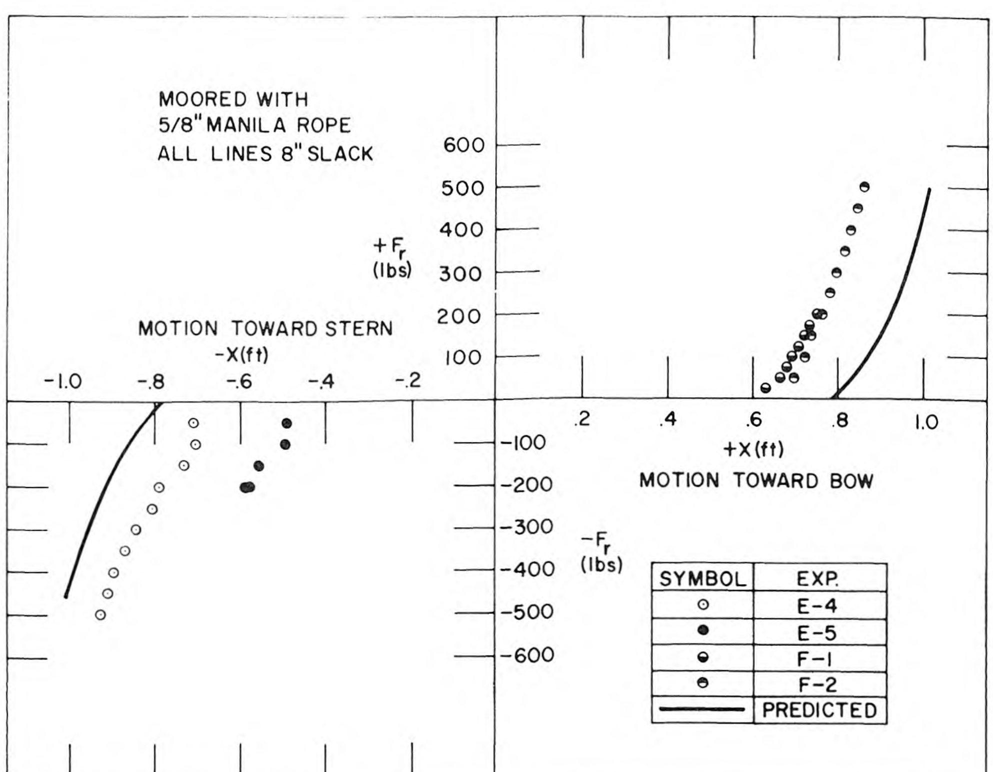

Fig. 4.9. Measured and Predicted Restoring Force vs. Displacement: Harbor Boat No. 3, All Lines 8 Inches Slack

boat obtained with the boat moored in a number of different ways are considered to be correct since these periods are not affected by the somewhat arbitrary determination of the at-rest-position. Since the objective of these prototype measurements was to evaluate these periods, the limitation of the measurements which has been discussed is not considered to be serious.

The predicted force-displacement curves for the cases of slack mooring are presented in Figs. 4.8 and 4.9 along with the measured data. The agreement between the measured and the predicted relations is poor primarily due to the difficulty in determining the at-rest-position as described previously. However, in both cases the slope of the predicted and measured curves are in agreement which indicates that although the free travel may not be predicted properly the method predicts the elastic characteristics of the lines reasonably well.

The predicted force-displacement curves for the three cases are combined and presented in Fig. 4.10 for the bow lines and the stern lines; these curves are the same as those shown in Figs. 4.7, 4.8 and 4.9. Fig. 4.10 shows that only the force-displacement curves for the case of taut lines can be approximated by an expression of the form: $F_r = Kx^n$; as soon as slack is introduced into the lines such an expression does not fit the data well.

Another interesting feature is seen in the results presented in Fig. 4.10. Due to the different mooring geometry for the bow lines and the stern lines, the former represents a spring system which is not as stiff as the latter. Hence, the restoring force of the moored

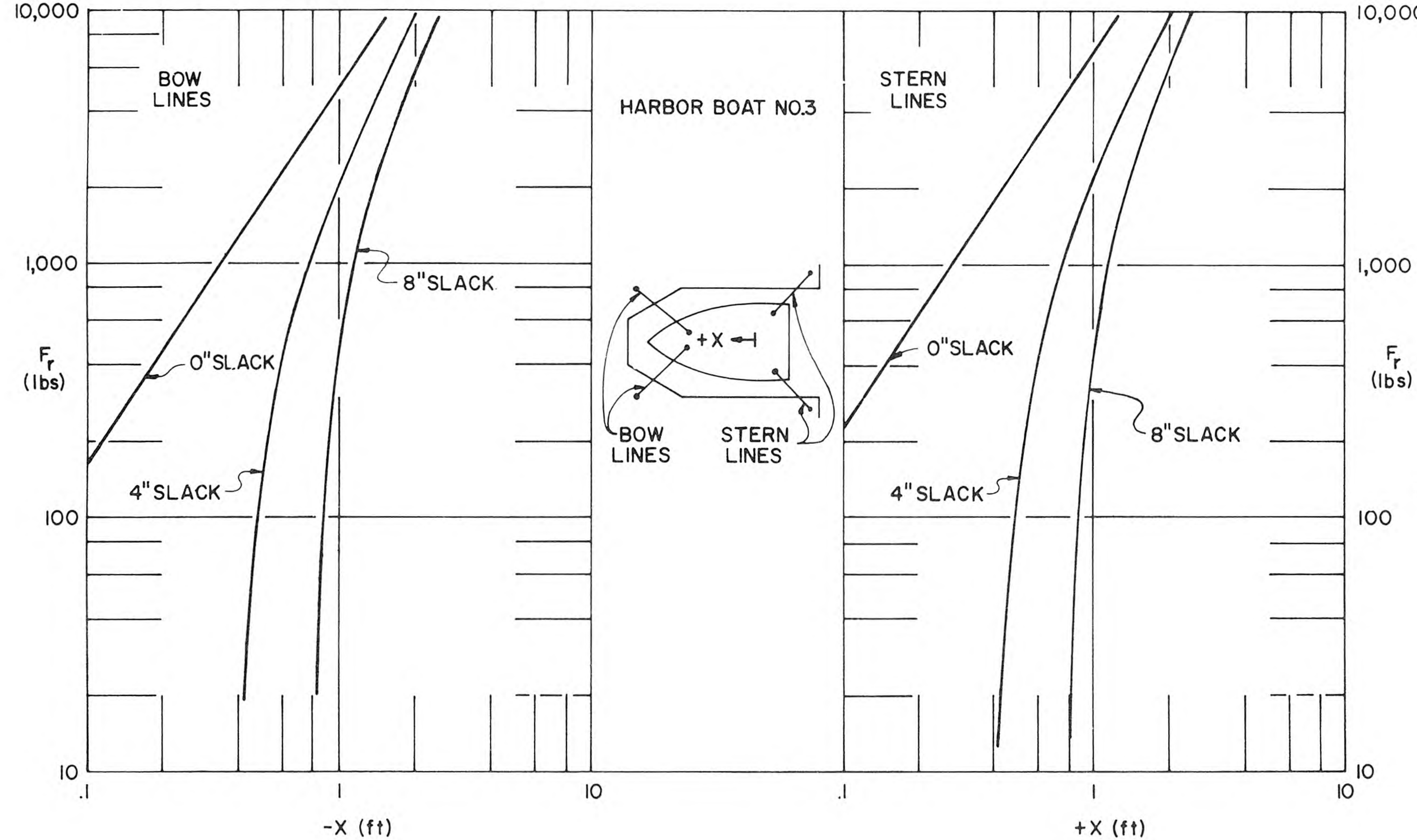

Fig. 4.10. Predicted Restoring Force vs. Displacement, Harbor Boat No. 3, All Lines: Taut, 4 Inches Slack, 8 Inches Slack

boat would be non-linear and asymmetrical similar to that shown in Fig. 2.2e. However, when slack is introduced, this asymmetrical effect is not as apparent, since it is masked by the free travel of the boat which is associated with the line slack.

4.4 Measured and Predicted Periods of Free Oscillation

As described in Section 4.2, in order to measure the period of free oscillation of the moored boat it was initially displaced in either the direction of the bow or the stern and then released with time-lapse moving pictures taken of the resultant motion. The film which was obtained in this way was analyzed in the laboratory on a film-frame analyzer. Employing this method each frame is projected sequentially and the displacement of the scale mounted on the boat determined for each frame relative to the scale location when the boat is in the averaged at-rest-position (see Section 4.3).

The free oscillations of Harbor Boat No. 3 moored with taut lines are shown in Figs. 4.11a and 4.11b. Fig. 4.11a shows the displacement-time history for the boat when a force of 500 lbs is applied to the bow by 4.11b presents similar data for a force of 500 lbs applied to the stern. In both cases the effect of damping is obvious. The periods of oscillation were obtained from these records by measuring the time from a maximum to the following minimum and averaging these over a number of cycles and multiplying the result by two. Thus the average period of oscillation for the boat displaced from the bow is 2.60 sec and from the stern it is 3.18 sec. If it had been possible to increase the applied force one would expect the period of oscillation to decrease.

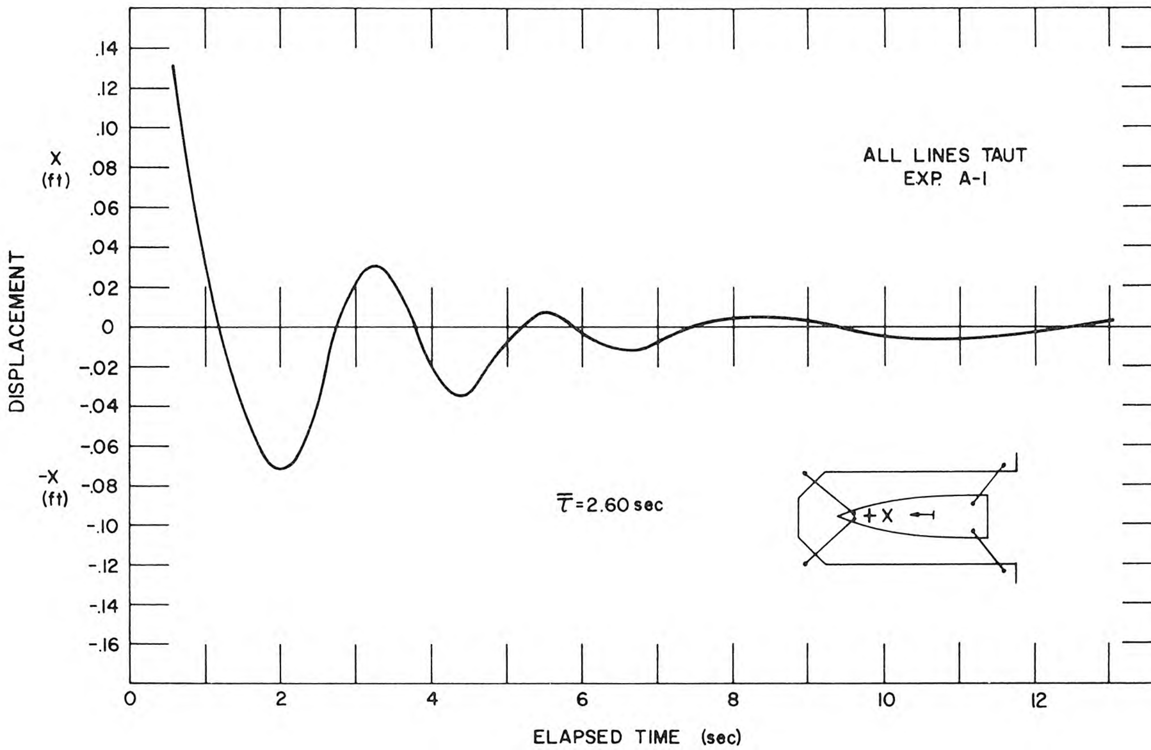

Fig. 4.11a. Free Oscillations, Harbor Boat No. 3: Taut Lines, Bow Displacement

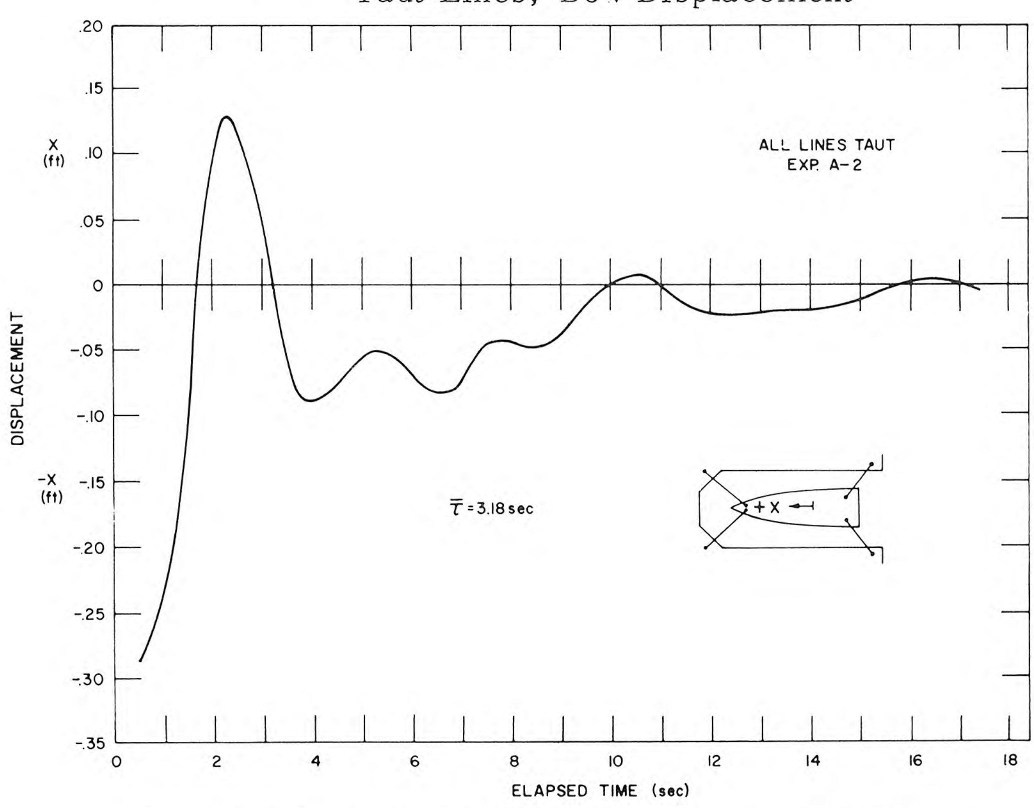

Fig. 4.11b. Free Oscillations, Harbor Boat No. 3: Taut Lines, Stern Displacement

Similar tests were conducted with 4 inches of slack in all lines. The results of these tests are shown in Figs. 4.12a and 4.12b where the displacement is also plotted as a function of time. In both cases, a 500 lb load applied to the bow (Fig. 4.12a) and a 500 lb load applied to the stern (Fig. 4.12b), the period of oscillation was approximately the same: 6.2 sec for an initial deflection toward the bow and 5.6 sec for an initial deflection toward the stern. Damping is also in evidence in these two cases.

Figs. 4.13a and 4.13b show typical examples of the free oscillations with 8 inches of slack introduced into all mooring lines. In both cases the important boat motions, from the point of view of the periods of free oscillation, occur within 50 sec of release. Fig. 4.13a shows the resulting motion after the boat had been displaced by a 500 lb force applied to the bow and Fig. 4.13b is for the case of a 200 lb load applied to the stern. The average period of oscillation for the former case is 18.34 sec and for the latter case it is 22 sec. These two cases show how the magnitude of the applied force can affect the period of oscillation, i.e., increasing applied loads result in decreasing periods of oscillation. Since in most dynamic problems damping affects the magnitude of the motion to a much greater extent than it affects the period of the oscillation the effects of viscous damping which are in evidence in all of the results are not considered important with relation to the main objective of this study. Figs. 4.11, 4.12, and 4.13 have been presented to indicate the form of the free oscillations; the periods of oscillation obtained from all tests are summarized in Table 4.3. (In all cases except one, for a given mooring condition the period of free oscillation decreases as the applied load increases.

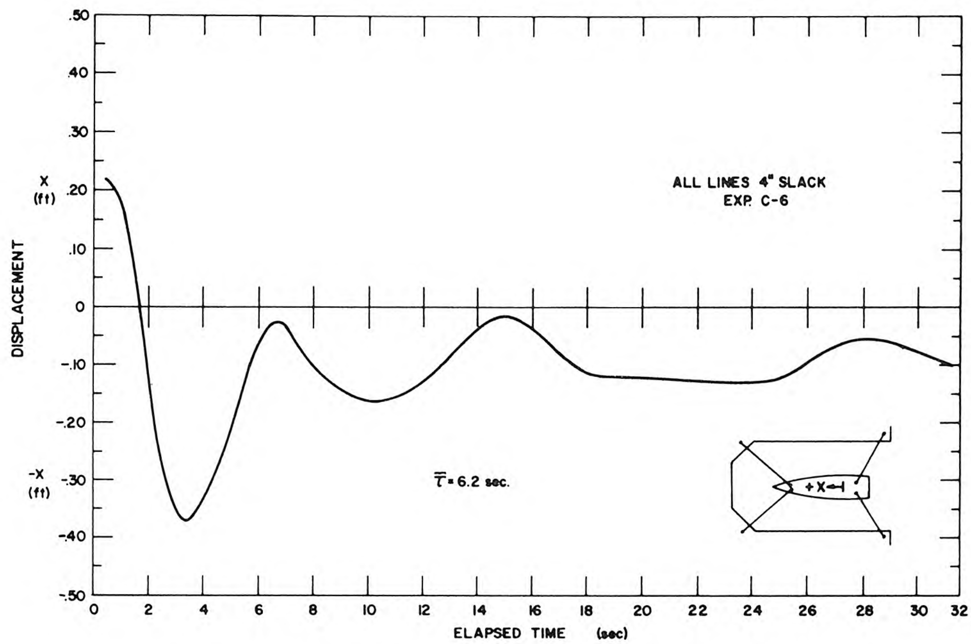

Fig. 4.12a. Free Oscillations, Harbor Boat No. 3: 4 Inches Slack, Bow Displacement

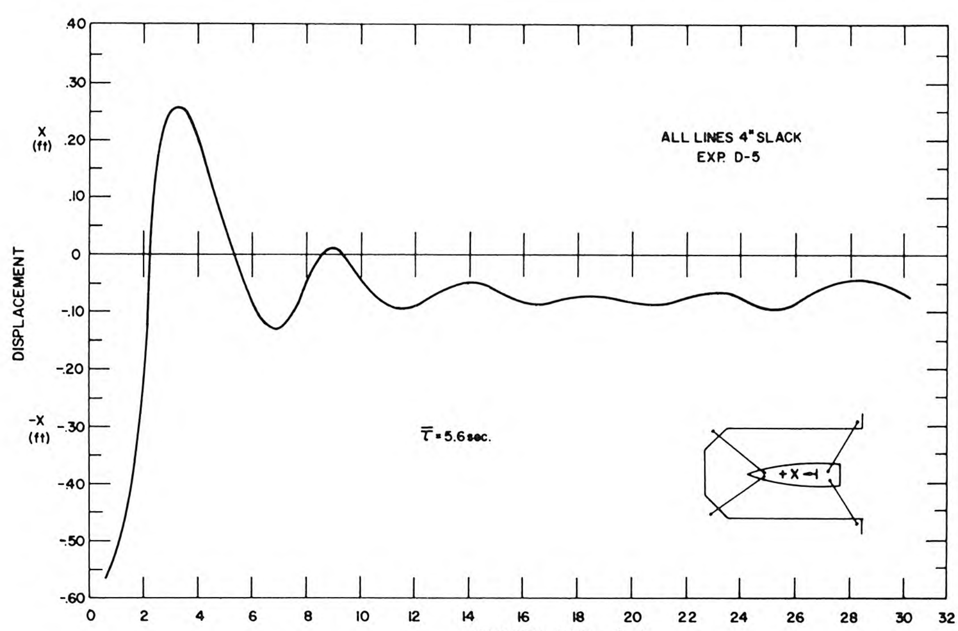

Fig. 4.12b. Free Oscillations, Harbor Boat No. 3: 4 Inches Slack, Stern Displacement

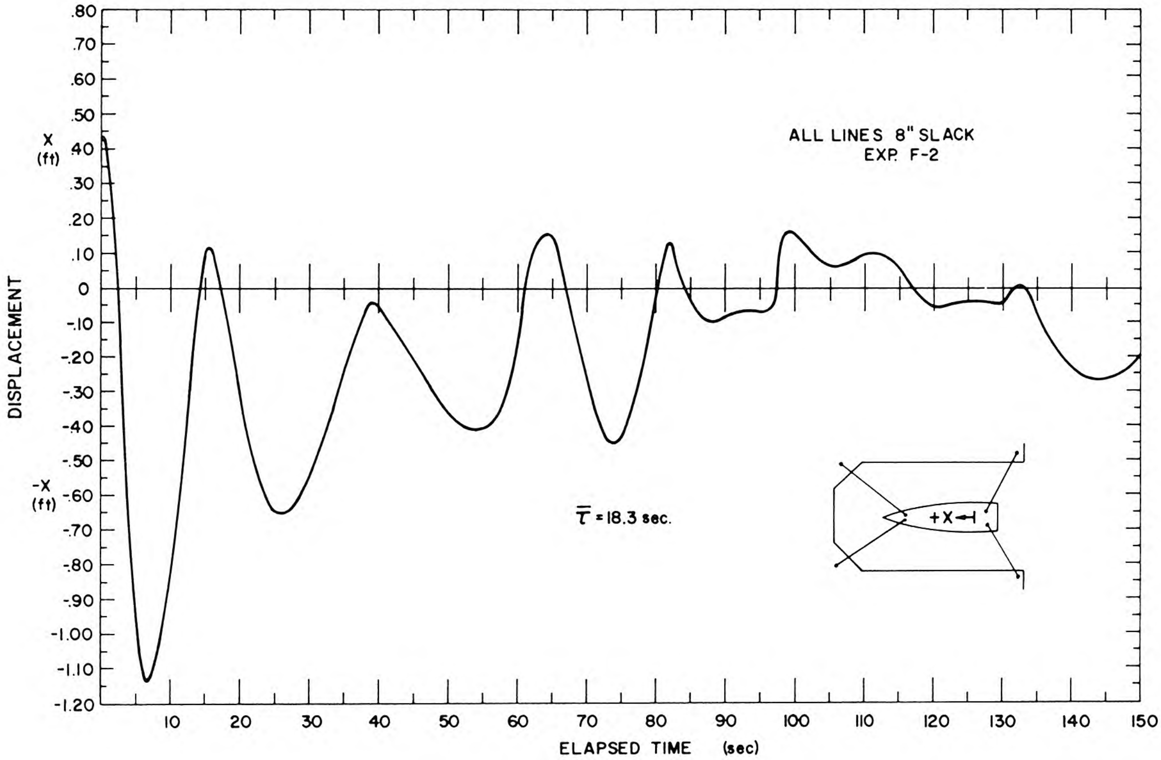

Fig. 4.13a. Free Oscillations, Harbor Boat No. 3: 8 Inches Slack, Bow Displacement

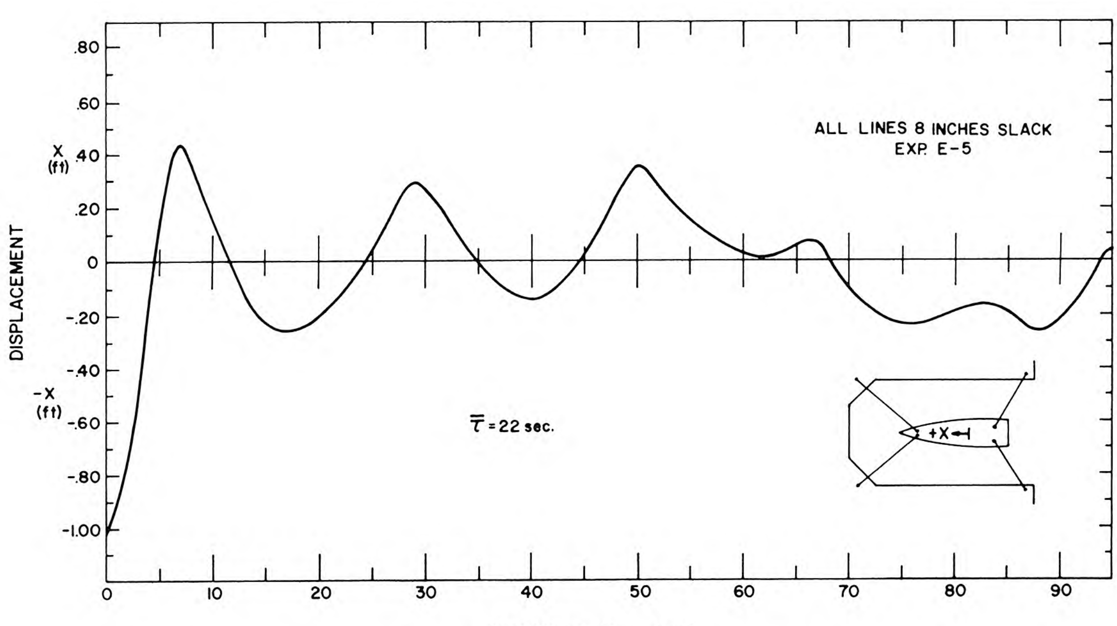

Fig. 4.13b. Free Oscillations, Harbor Boat No. 3: 8 Inches Slack, Stern Displacement

Table 4.3. Periods of Free Oscillation of Harbor Boat No. 3

Experiment No.	Applied Force		Line Condition	Period of Oscillation (secs)
	To Bow (lbs)	To Stern (lbs)		
A-1	500		Taut	2.6
A-2		200	Taut	2.28
A-2		500	Taut	3.18
C-5	200		4 in. slack	7.4
C-6	500		4 in. slack	6.2
D-4		200	4 in. slack	7.4
D-5		500	4 in. slack	5.6
F-1	200		8 in. slack	19.9
F-2	500		8 in. slack	18.3
E-5		200	8 in. slack	22.0
E-4		500	8 in. slack	15.9

The results of these experiments have been compared to the results of the analytical study which was described in Section 2. It is noted that the condition of free oscillations is a special case of Eq. 2.24 when ζ (the maximum water particle acceleration averaged over the displaced volume of the body) is zero, i.e., the force acting on the moored boat is zero. In a similar way Eq. 2.38, which describes the dynamic response of the boat, is simplified and the relation between the initial boat displacement and the period of oscillation can be determined for the three mooring systems under consideration: taut lines (that is, zero slack in all lines), 4 inches slack in all lines, and 8 inches slack in all lines. The curves which result are presented in Fig. 4.14 as the

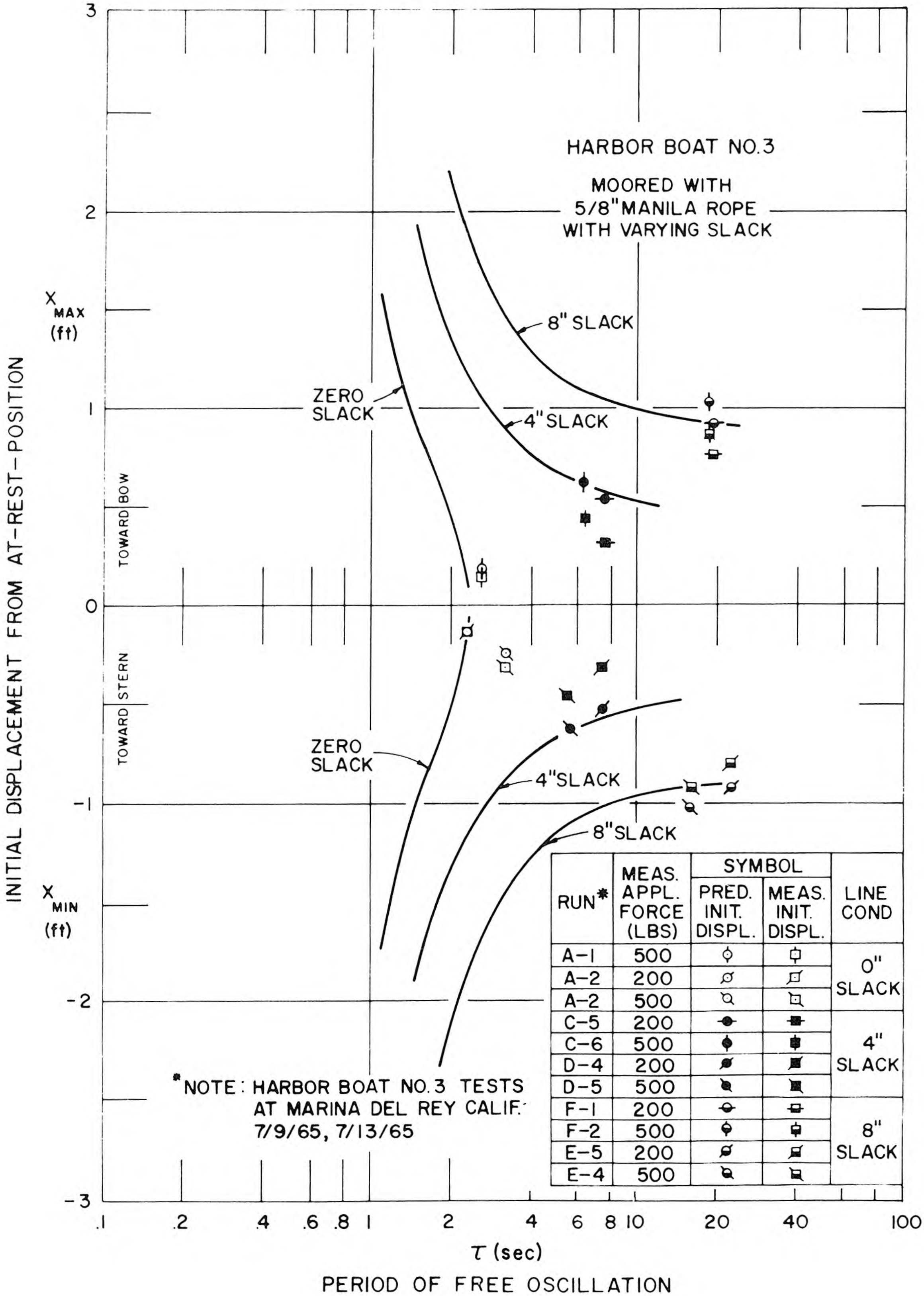

Fig. 4.14. Measured and Predicted Periods of Free Oscillation: Harbor Boat No. 3

variation of the period of free oscillation with the initial displacement in either the positive or negative x-direction. It is seen that the mooring system which is described by the dimensions presented in Table 4.2 must be nearly symmetrical, since any asymmetry due to different lengths of lines is exhibited in the free oscillations only for the case of taut lines.

In order to solve Eq. 2.38 for the period of the free oscillations for a particular initial displacement it is first necessary to define the mass M as well as the virtual mass coefficient C_M. As discussed previously the mass is based upon a displaced weight of Harbor Boat No. 3 of 7000 lbs. The virtual mass coefficient is more difficult to define, since it is known to be a function of both the shape of the body and the frequency of oscillation as well as other dimensions of the berthing arrangement (see Raichlen (1965)). Eq. 2.38 shows that for the case of free oscillations the period of the oscillations is inversely proportional to the square root of the virtual mass coefficient. Therefore, to some extent the oscillations are insensitive to this parameter if a reasonable estimate is made. This does not mean that the value of C_M is unimportant, but if the C_M used is within say ± 10% of the exact value then the period of oscillation will be within ± 5% of the true period. A value of the virtual mass coefficient, C_M, of 1.2 has been used for Harbor Boat No. 3 which corresponds to data from Wilson (1950) for a floating body with a similar beam-to-length ratio.

The influence of line slack on the free oscillations is interesting. Consider a given initial displacement; as the slack increases the period of oscillation increases significantly. This is because the restoring force is active only over a portion of the range of the boat motion. For the range of motion within the region of free travel the boat must move at a relatively constant velocity; hence, referring to Fig. 4.14, as the slack increases the free travel and the time it takes the boat to traverse this distance increases.

The data presented in Table 4.3 are also included in Fig. 4.14. Two sets of data are shown in Fig. 4.14: one set where the initial displacements are determined from the applied load and the experimentally determined restoring forces (either Fig. 4.7, 4.8, or 4.9) and one set where the initial displacements correspond to those obtained from the applied loads and the theoretically determined restoring force relations presented in Fig. 4.10. Both cases show reasonable agreement with the predicted periods of free oscillation except for the case of 4-inch slack in all lines and the case for taut lines with a force of 500 lbs applied in the negative x-direction. It should be noted that for small values of displacement the theoretical curve of free oscillations for the case with taut lines tends to a constant value of the period of oscillation. This is because of the approximation used to describe the restoring force (Eq. 2.33) which becomes linear for small x and it is not due to the nature of the mooring system. For the cases with slack, no curve is shown for small values of displacement, since for values of $X < \Delta_f$ (displacements less than the free travel) there is no associated period of oscillation because the restoring force is zero.

The initial displacements obtained from the measured force-displacement curves generally are smaller than those obtained from the predicted force-displacement curves. In general this can be attributed to the fact that Eq. 2.51 predicts somewhat greater free travel than measured during the tests. This difference could be caused by inaccuracies in measuring the dimensions reported in Table 4.2 or by the difficulties described previously in measuring the free travel of the boat. Nevertheless, considering all of the problems which are inherent to prototype testing the agreement between the experiments and the theory with respect to the periods of free oscillation is considered to be reasonable confirmation of the analysis presented in Section 2. Therefore, it is felt that this analysis can be used with some confidence in predicting the important range of periods of oscillations of moored small boats.

4.5 Predicted Response Curves

The non-linear response curve for Harbor Boat No. 3 moored with taut lines is presented in Fig. 4.15 as the variation of the maximum displacement of the boat in surge from the at-rest-position, X_{max} and X_{min}, with the wave period, T, for constant values of the forcing function, ζ. The curve for $\zeta = 0$ is the same as the one presented in Fig. 4.14 for the free oscillations and is called the "backbone" curve for non-linear response curves. Since damping is neglected energy is not dissipated and hence the curves do not approach a finite maximum displacement; the region where these curves tend to an infinite displacement is termed resonance.

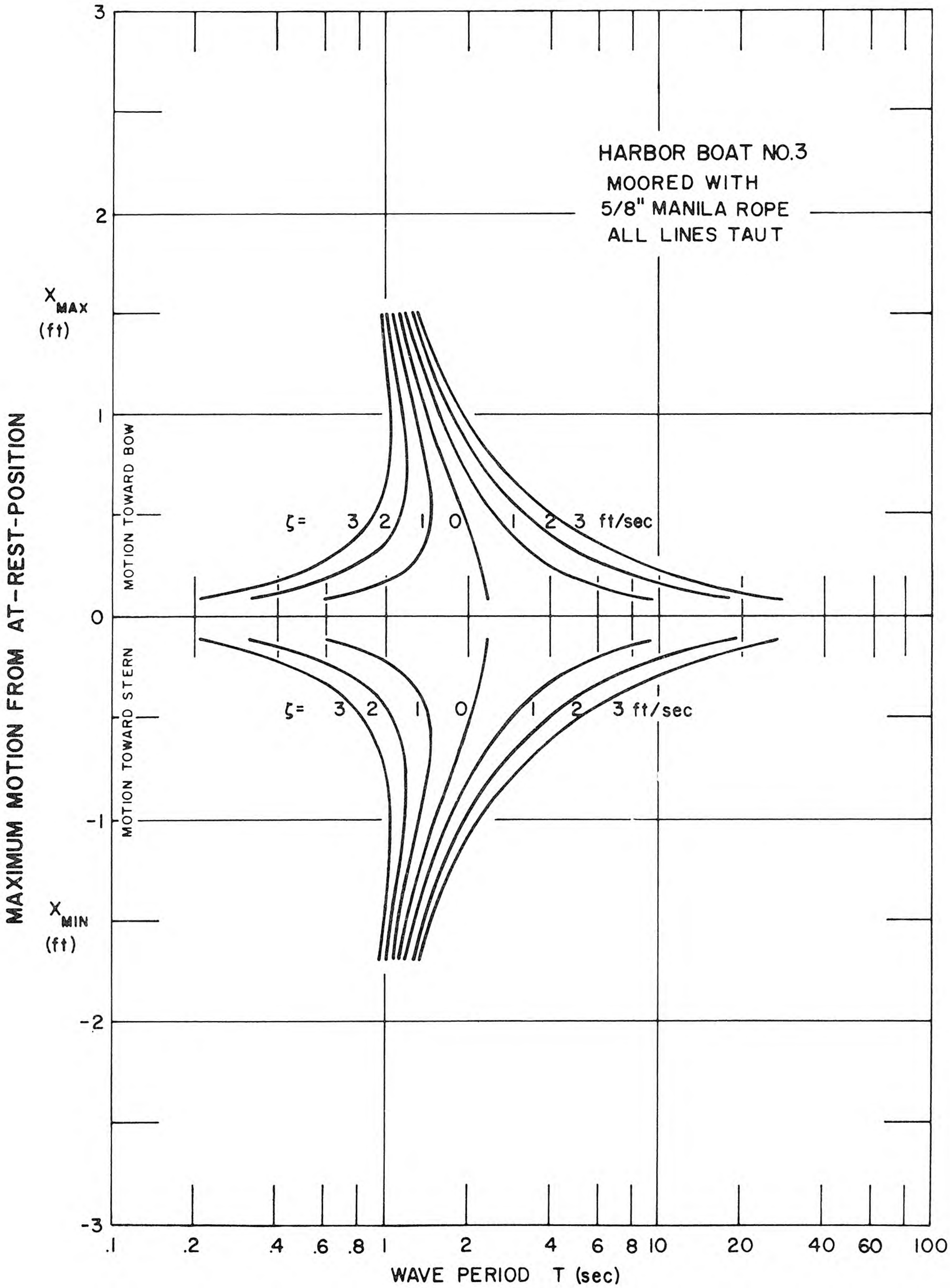

Fig. 4.15. Response Curve: Harbor Boat No. 3, Taut Lines

These response curves have been obtained from Eq. 2.38 in accordance with the assumptions presented in Section 4.4. The restoring force-displacement curves presented in Fig. 4.10 have been used in the analysis. It is noted from Eq. 2.38, which describes the non-linear undamped response of the boat, the boat displacements X_{max} and X_{min} (or X and A_1) are functions of the wave period T and the forcing function ζ. Therefore, in order to graphically present these curves one of the latter two parameters must be kept constant. In this report the response curves are determined for constant values of the parameter ζ with X_{max} and X_{min} and T variable.

It has been seen from Eq. 2.17c and 2.11a that ζ is simply the maximum with respect to time of the water particle velocity averaged over the displaced volume of the moored body. This in turn is a function of the standing wave amplitude as well as certain parameters which deal with the geometry of the boat and the berth. Therefore, the interpretation of ζ is not simple and its value must be determined for fixed system parameters and variable wave period and amplitude. This will be discussed in detail later in this section.

Certain features of the response curves presented in Fig. 4.15 are interesting as they relate to the general problem of the dynamics of small moored boats. Consider first the case where the wave period is contant, but the wave amplitude increases with time to a maximum. For the example chosen consider a wave period of 1.2 sec and let the value of ζ increase from say 1.0 ft/sec to 2.0 ft/sec, indicating for the case shown in Fig. 4.15 a doubling of the wave amplitude. (It is realized that this is a physically unrealistic wave period, but it will serve to illustrate one aspect of non-linear response curves.) As the forcing function ζ increases to 1.0 ft/sec a maximum displacement of approximately X_{max} = 0.2 feet is reached. If the wave amplitude is increased further until a value of ζ = 2.0 ft/sec is reached it is noted that the maximum boat displacement is approximately 0.7 feet. However, this occurs on a portion of the response curve which is unstable, i.e., the slope of the curve at this point expressed as dX_{max}/dT is infinite. Therefore, the maximum boat deflection will "jump" to the higher more stable branch of the curve for ζ = 2 ft/sec which would be at a displacement of approximately 1.6 feet or a sudden increase in movement of over twofold. Due to this jump the line stress could go from a completely safe value to a value which exceeds the breaking strength of the rope. This same type of jump can occur if the magnitude of the forcing function ζ remained constant but the wave period changed. (Since ζ is a function of wave period the wave amplitude would have to change accordingly.) Consider a value of ζ = 1 ft/sec and a wave period which increases continuously from 1 sec to 10 sec. Fig. 4.15 shows that for T = 1 sec the maximum boat displacement would be approximately 0.2 feet. When the period reaches a value T = 1.45 sec the slope of the response curve becomes infinite (X_{max} = 0.5 feet) and the displacement jumps to 1.1 feet on the upper branch of the curve. As the wave period increases further the maximum displacement will now decrease monotonically until it reaches a value of approximately 0.1 feet at a wave period of 10 sec. If, on the other hand, the line system had consisted of linear springs the response curve would have been described by Eq. 2.21 or Eq. 2.22. In that case for wave periods less than the natural frequency of the moored body the response would increase gradually to a maximum as the period increased and then decrease monotonically for periods greater than the resonant period.

Two other response curves are presented for Harbor Boat No. 3: Fig. 4.16 for the case of 4 inches slack in all lines, and Fig. 4.17 for the case of 8 inches slack in all lines. As one would surmise from Fig. 4.14, as the slack increases the family of response curves shifts to larger values of the wave period. This can be seen in Figs. 4.15, 4.16 and 4.17 by considering the range of important wave periods for say a 1-foot displacement. For the case of taut lines, considering a value of ζ = 3 ft/sec the range of wave period for a 1-foot maximum displacement is from approximately 1 sec to 2 sec. When the line slack is increased to 4 inches this range changes from 1.5 sec to 5 sec. For the case shown in Fig. 4.17 where there is 8 inches of slack in all lines the important range of wave periods is 2 sec to 26 sec.

These examples demonstrate that simple changes in the mooring system can have a profound effect upon the dynamics of the moored boat. For instance, the motions induced by storm waves (8 sec to 12 sec periods) for the boat moored with 8 inches of slack in the lines are approximately 5 to 6 times greater than those for the boat moored with taut lines. Therefore, whereas one mooring condition may be considered safe, the other may be potentially dangerous. In connection with this, the damage criterion for the boat moored with taut lines may

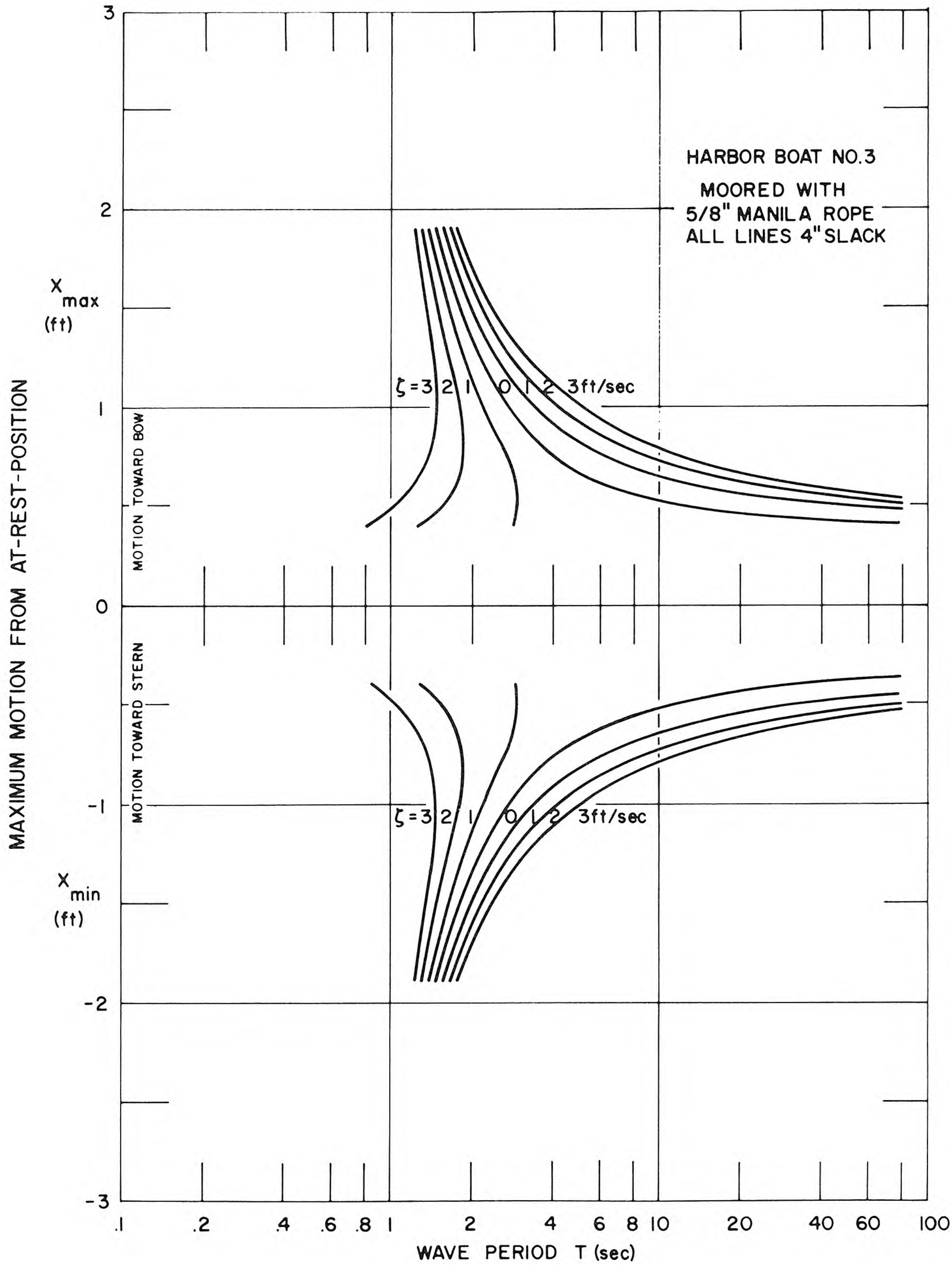

Fig. 4.16. Response Curve: Harbor Boat No. 3, 4 Inches Slack

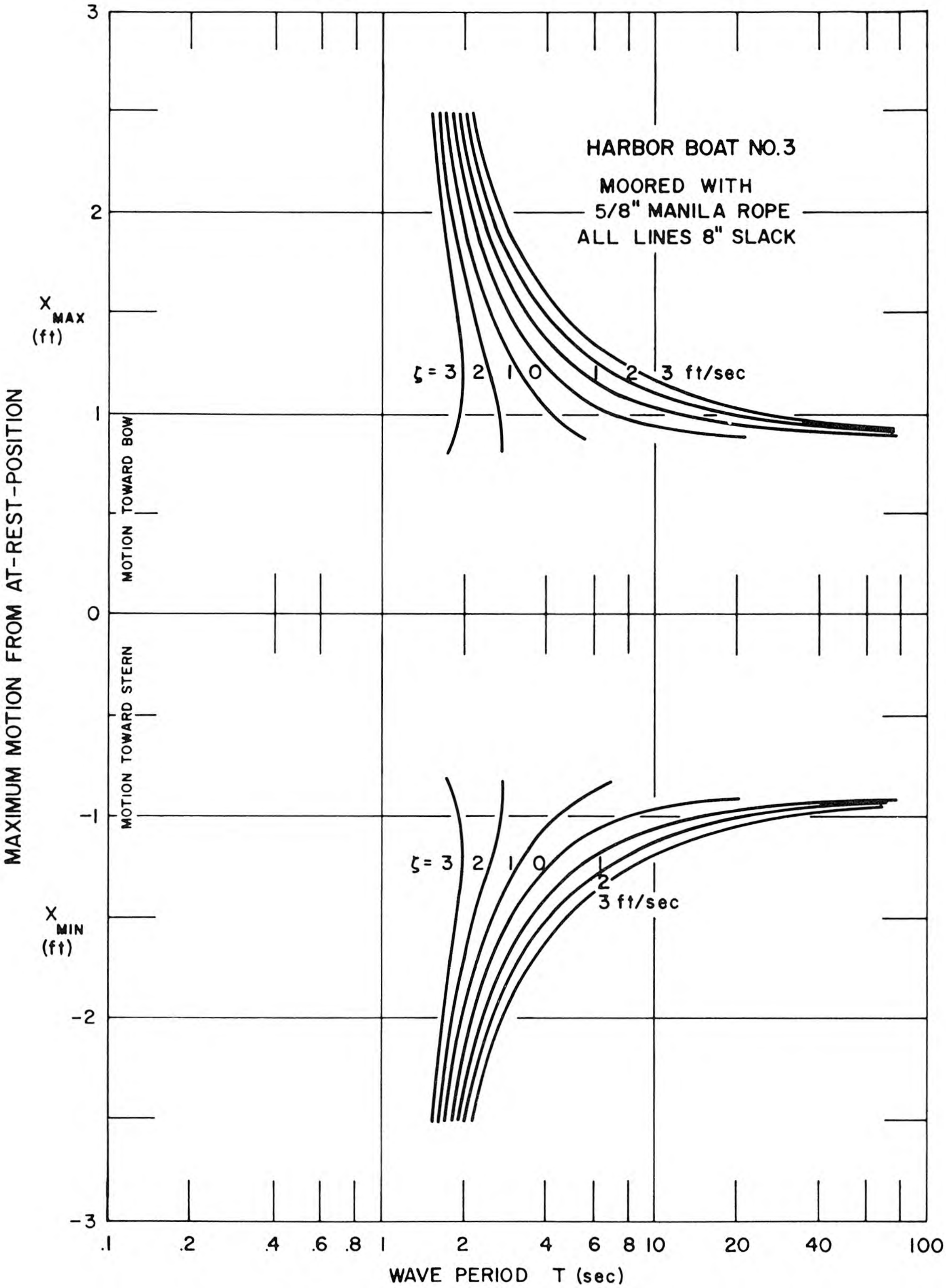

Fig. 4.17. Response Curve: Harbor Boat No. 3, 8 Inches Slack

be based upon failure of the lines or fittings; however, for the case with line slack it may be necessary to base a damage criterion upon the possible impact of the boat with the dock due to excessive motion.

Consider the variation of the forcing function, ζ, with wave period. A specific case is presented in Fig. 4.18 where the variation of the ratio ζ/A is shown with wave period for a block body with the dimensions of $2L = 18$ feet, $D = 0.68$ feet and the center of the body located 120 feet from a reflecting surface ($b = 120$ feet). (These dimensions are the block-body approximations to the boat-lines shown in Fig. 4.3.) The depth of water at the mooring site chosen for this example is 10 feet ($d = 10$ feet). These dimensions were chosento demonstrate some of the features of the forcing function and only in a general way do they represent conditions at the berth at Marina del Rey. However, this example will show how the response curve is a function of the geometrical configuration of the vessel and berth as well as the characteristics of the mooring system.

Fig. 4.18 shows the periodic nature of the ratio of the forcing function ζ to the standing wave amplitude A, i.e., ζ/A, with wave period; at certain intervals this function and the locus of the maxima of this function become zero. In other words, for the geometry chosen the forcing function and therefore the boat displacement becomes zero at certain wave periods due to the trigonometric terms in Eq. 2.17c becoming zero. Conversely the maxima are associated with these terms becoming a maximum. Physically the zeroes occur when the crest of the standing wave is located at the center of the body. For

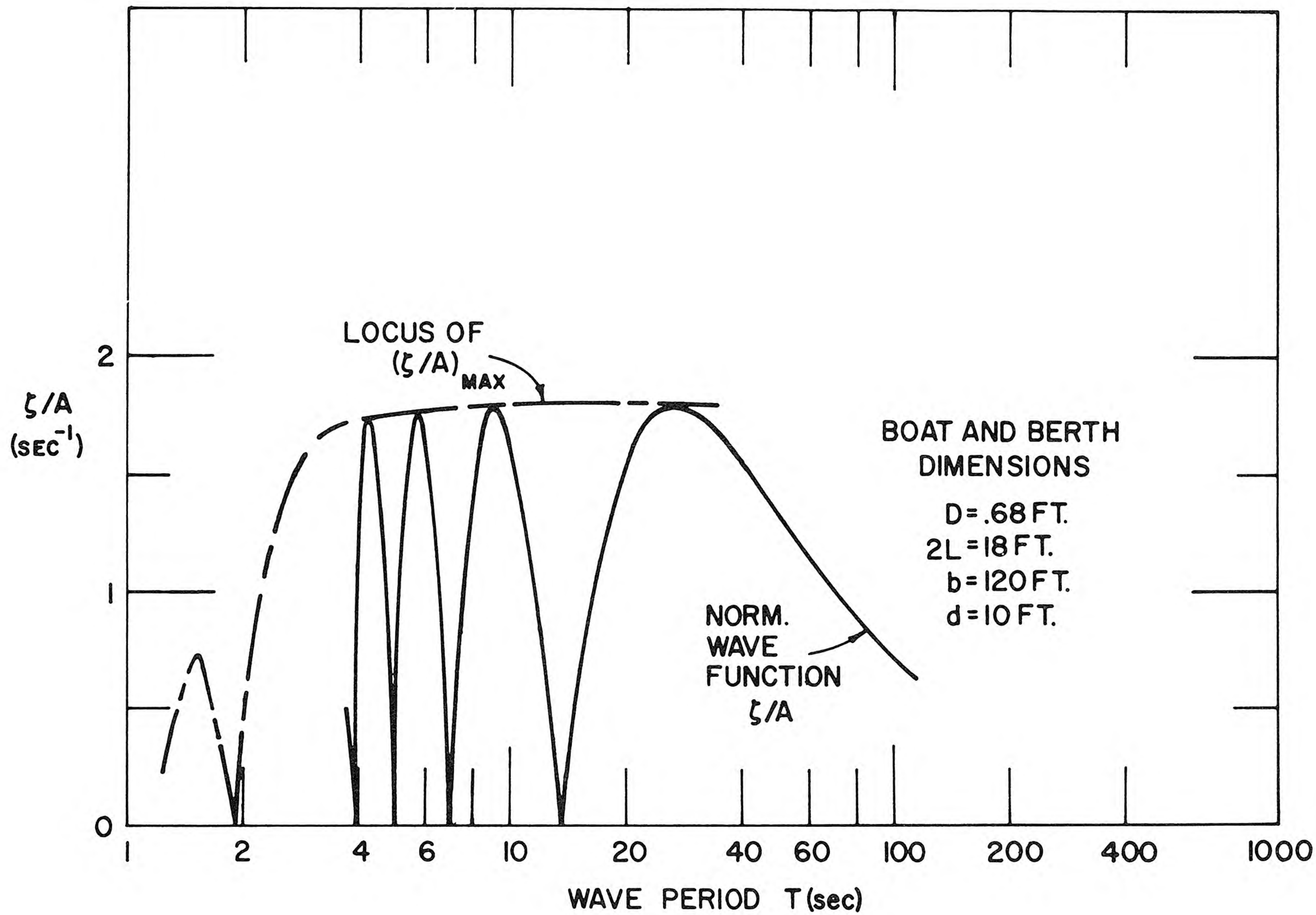

Fig. 4.18. Variation of Normalized Forcing Function, ζ/A, as a Function of Wave Period, T

this condition there is no net force acting on the body at any time and therefore no resulting body motion. The opposite is true at the periods associated with the maxima. For these wave periods a node occurs at the center of the body and a maximum longitudinal force acts on the vessel. Therefore, for example, the zero of the locus of maxima at $T = 1.9$ sec in Fig. 4.18 occurs when the wave length is equal to the length of the body ($kL = \pi$); hence, no net driving force is provided by the wave. The zeroes for $T > 1.9$ sec are associated with $kb = n\pi$ and the wave length is equal to $\frac{2}{n}b$ where $n = 1, 2, 3,$ etc. As the distance b increases the number of zeroes would also increase for a fixed range of wave period.

When the variation of the forcing function with wave period is introduced into a response curve such as Fig. 4.17 the response is no longer as simple as that described previously, i.e., following a constant value of ζ as the wave period changes. Instead, as the wave period changes, for a constant wave amplitude, the value of ζ varies and therefore the change in the maximum displacement of the boat becomes a function of both the mooring dynamics and the variation of the forcing function. However, as a first approximation to the maximum boat displacement the locus of maxima such as shown in Fig. 4.18 can be used. For example, in that case the maximum is approximately ($\zeta/A = 1.8$ sec^{-1}; hence, for a standing wave with a 1-foot amplitude at worse the response would follow a curve of constant ζ of 1.8 ft/sec.

It should be obvious from this discussion that since it is not possible to express the response curves for a particular boat simply as families of curves of constant wave height, the variation of the forcing function ζ with wave period must be investigated to determine the appropriate family of curves to be used for a particular standing wave height and a particular mooring geometry. As suggested, this approach may be simplified by obtaining the maximum value of the forcing function ζ from the locus of maxima for the wave height of interest, as shown in the example of Fig. 4.18, and then studying the variation of the appropriate response curve with wave period to determine if a particular boat would be in danger in a marina due to a certain incident wave system.

5. SOME ADDITIONAL PROTOTYPE INVESTIGATIONS

In an effort to evaluate the characteristics of some of the mooring systems presently in use, a limited study was conducted of boats moored at Marina del Rey near Los Angeles, California. This was the same small craft harbor that was used for the prototype tests described in Section 4. The objective was to gather enough information in the field on the dimensions of mooring systems in use so that reasonable estimates could be made of the response of these small boats to standing waves. In this way it was felt that a first approximation could be made in determining the important range of wave periods of a small boat harbor. Indeed Marina del Rey is not a typical small boat harbor due to its size; however, the boats and the mooring systems used do represent reasonably general systems.

5.1 General Considerations

An important consideration in determining the range of wave periods and associated wave heights which could cause damage in a small craft harbor is the frequency distribution of boat sizes (and associated displaced weights) which could be expected for various harbors. Unfortunately this type of information was not readily available for Marina del Rey; however, data have been obtained for the distribution of pleasure boats moored in Long Beach Marina in Long Beach, California. These data were obtained in private communication from the Los Angeles District of the U.S. Army Corps of Engineers and they are presented in Fig. 5.1 as the percentage of boats which were less than a particular size at the time of their investigation. The class intervals of length chosen in that study was 5 feet starting

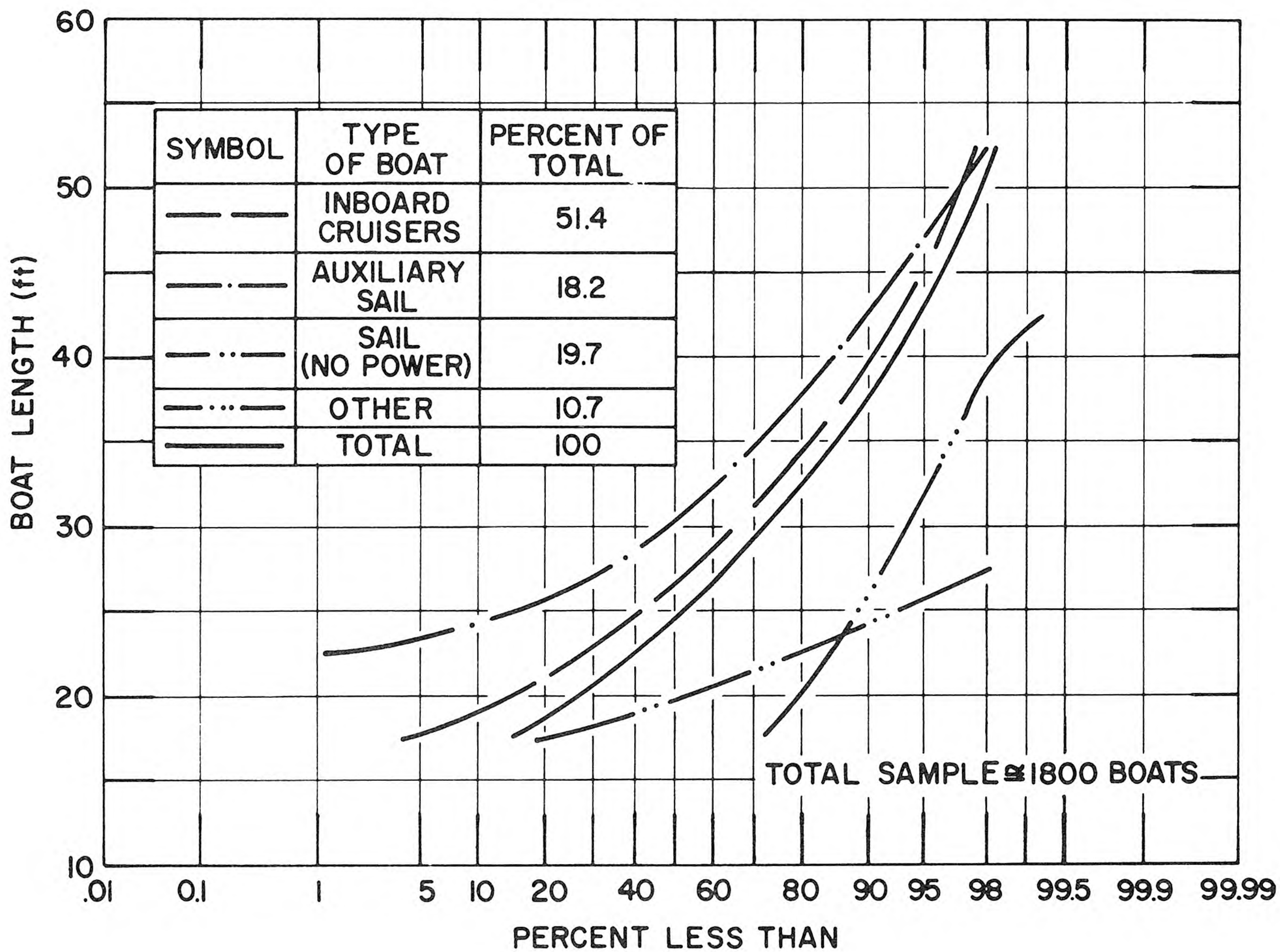

Fig. 5.1. Frequency Distribution of Lengths of Pleasure Boats in Long Beach Marina, Long Beach, Calif. (September 1966).

with a length of 15 feet. Five frequency distributions are presented in Fig. 5.1: cruisers with inboard motors, sailboats with auxiliary power, sailboats without power, boats which fall into none of these three categories (labeled in Fig. 5.1 as other), and the distribution by length of the total sample of about 1800 boats.

It should be emphasized that these distributions are for one small craft harbor surveyed at one particular time; however, it is felt that the gross characteristics of the curves presented provide a guide to important boat sizes in a marina.

Fig. 5.1 shows that the inboard cruisers represented 51.4 percent of the total sample and the median boat length was between 25 feet and 30 feet. In addition 75 percent of these cruisers were less than 30 feet to 35 feet long. Although sailboats with power represented only 18.2 percent of the total sample the median length of this class was somewhat larger, 30 feet to 35 feet with 75 percent less than 35 feet to 40 feet. Pure sailboats and "other" boats which comprise 30.4 percent of the total sample have median lengths which are less than approximately 20 feet. It is noted that the distribution of the total sample has approximately the same median and 75 percent length as that of the power cruisers because of the preponderance of inboard power cruisers in this sample. Therefore, the major attention in this portion of the study was directed toward inboard power cruisers. This type of boat generally has a modified "Vee" cross section similar to Harbor Boat No. 3 (see Fig. 4.3) and is more susceptible to surge motions than other classes, such as sailboats where the preferable motion may be in roll due to the large keel.

The information on the distribution of boat lengths in a marina are of interest primarily for the insight it provides in the expected distribution of small boat shapes and displaced weights in a small craft harbor. Some data on the variation of the approximate loaded weight with length of some small boats is presented in Fig. 5.2. Two curves are presented in Fig. 5.2: one which is the envelope of weights of power and sailboats from Chaney (1961) and one which represents the manufacturers information for seven boats built by Tollycraft of Kelso, Washington. Included in Fig. 5.2 are two weights for Harbor Boat No. 3; the smaller weight being the manufacturer's estimate of the unloaded weight and the 7000 lb weight (the estimated loaded weight) which was used in this study. With reference to Figs. 5.1 and 5.2 a median boat weight for power cruisers and the total population would be between approximately 5000 lb and 7000 lb. In addition approximately 75 percent of all boats in Long Beach Marina would have displaced weights less than about 9,000 lbs to 12,000 lbs. (Since this estimate is based on the data of Fig. 5.2, it is considered to be only approximate.)

5.2 Description of Boats and Their Mooring Systems

The mooring dimensions and the characteristics of the mooring systems were determined for seven small boats located at Marina del Rey. The boats which were chosen were inboard power cruisers whose lengths corresponded, as closely as possible, to a representative sample of the distribution presented in Fig. 5.1. The range of boat lengths for the sample was from 22 feet - 5 inches to 38 feet. Referring to Fig. 5.1, for Long Beach Marina only 28 percent of the

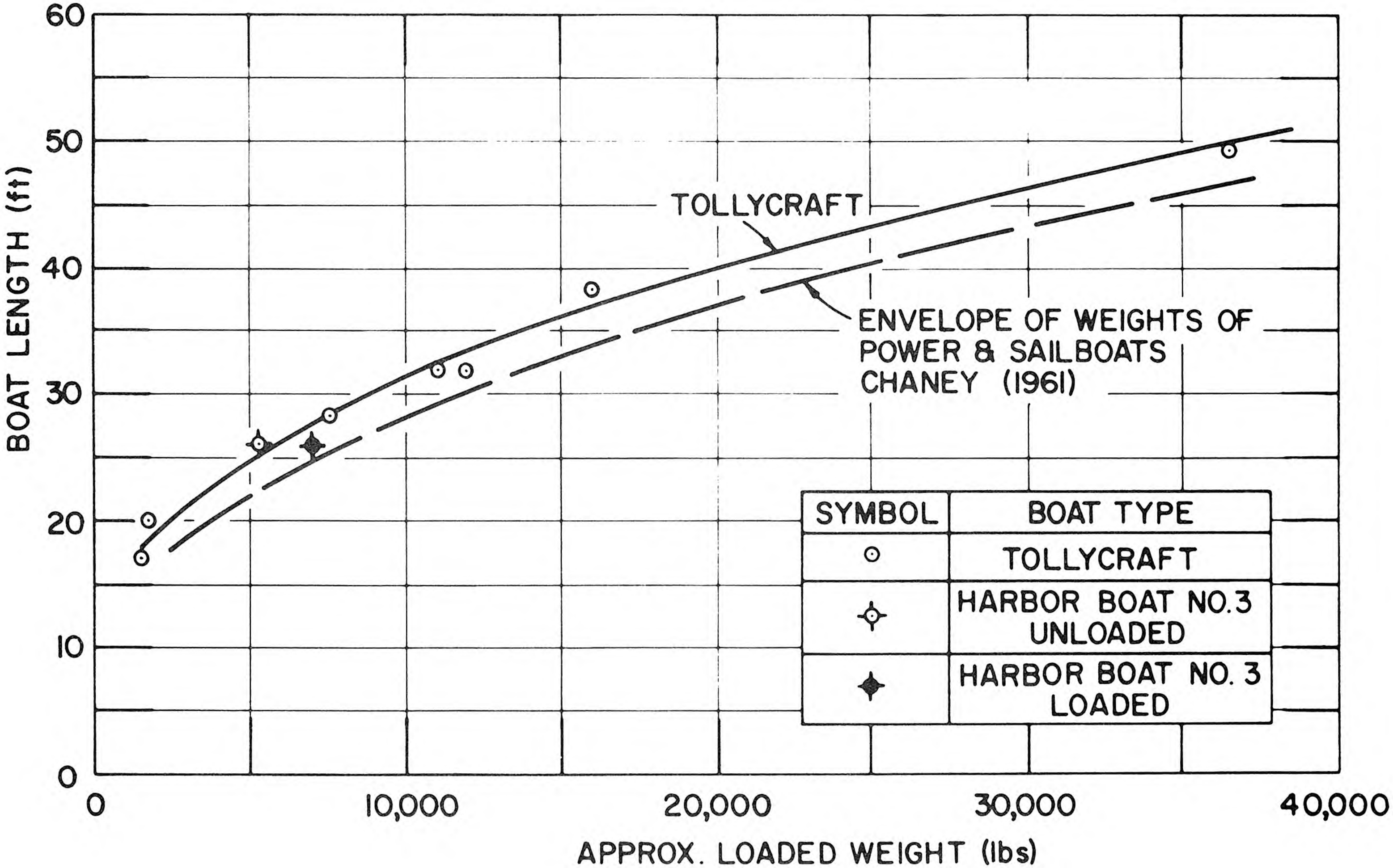

Fig. 5.2. Boat Length vs. Loaded Displaced Weight

power cruisers had lengths less than the former figure and 85 percent of the inboard power boats had lengths less than the latter figure. The lengths and estimated weights of the boats used in this study are presented in Table 5.1. The second column in this table gives the name of the company which built the particular boat along with the model as designated by the builder. The weights shown in Table 5.1 are estimated from Fig. 5.2 using the curve for boats built by Tollycraft instead of the curve which represents an envelope of data pertaining to weight presented by Chaney (1961).

Table 5.1. Measured Lengths and Estimated Weights of a Sample of Small Boats

State of California Registration Number	Boat Builder (Boat Type)	Length (Ft-In.)	Estimated Weight (Lbs)
CF 6141 CB	Chris-Craft (Cavalier)	22'-5"	3,700
CF 0675 CJ	Owens	24'-8"	5,200
CF 0394 CV	Tollycraft	25'	5,200
CF 0310 CV	Tollycraft	25'	5,200
CF 7651 CL	Owens (Flagship)	29'-3"	8,200
CF 7198 CJ	Owens (33 Sedan)	33'	11,500
CF 1756 CW	Chris Craft (38 Commander)	38'	17,000

Photographs of the boats described by Table 5.1 are presented in Figs. 5.3 through 5.9. These photographs show the mooring systems which are generally in use at Marina del Rey and indicate some of the major differences in the mooring systems for boats of different size. An example of the extremes can be seen in Fig. 5.3 and Fig. 5.9: these photographs are for the smallest and the largest boats in the sample investigated. In Fig. 5.3 the slip is much larger than is necessary for that particular boat; therefore, the mooring lines are atypically long. The 38 foot boat shown in Fig. 5.9 is large for the slip, hence the clearance between the boat and the slip is a minimum and the mooring lines are relatively short.

The relative size of the boat and the slip can change the action of the restoring force. For example, the stern lines restrain the boat in surge, for the boat shown in Fig. 5.3, when the motion of the boat is in the direction of the bow (positive x-direction) whereas for the boat shown in Fig. 5.9 the bow lines restrain the boat when the movement of the boat is toward the bow (positive x-direction). The latter type of restoring force system can increase the probability that restoring forces will be asymmetrical.

Actually the condition which causes the type of mooring described and shown in Figs. 5.7 and 5.9, for example, is generally one of economics. The rental charge for slips increases with their lengths, the boat owner, therefore, tends to rent the smallest possible slip for his boat. This condition makes mooring conditions such as those shown in Fig. 5.3 unusual in most small craft harbors. In attempting to fit

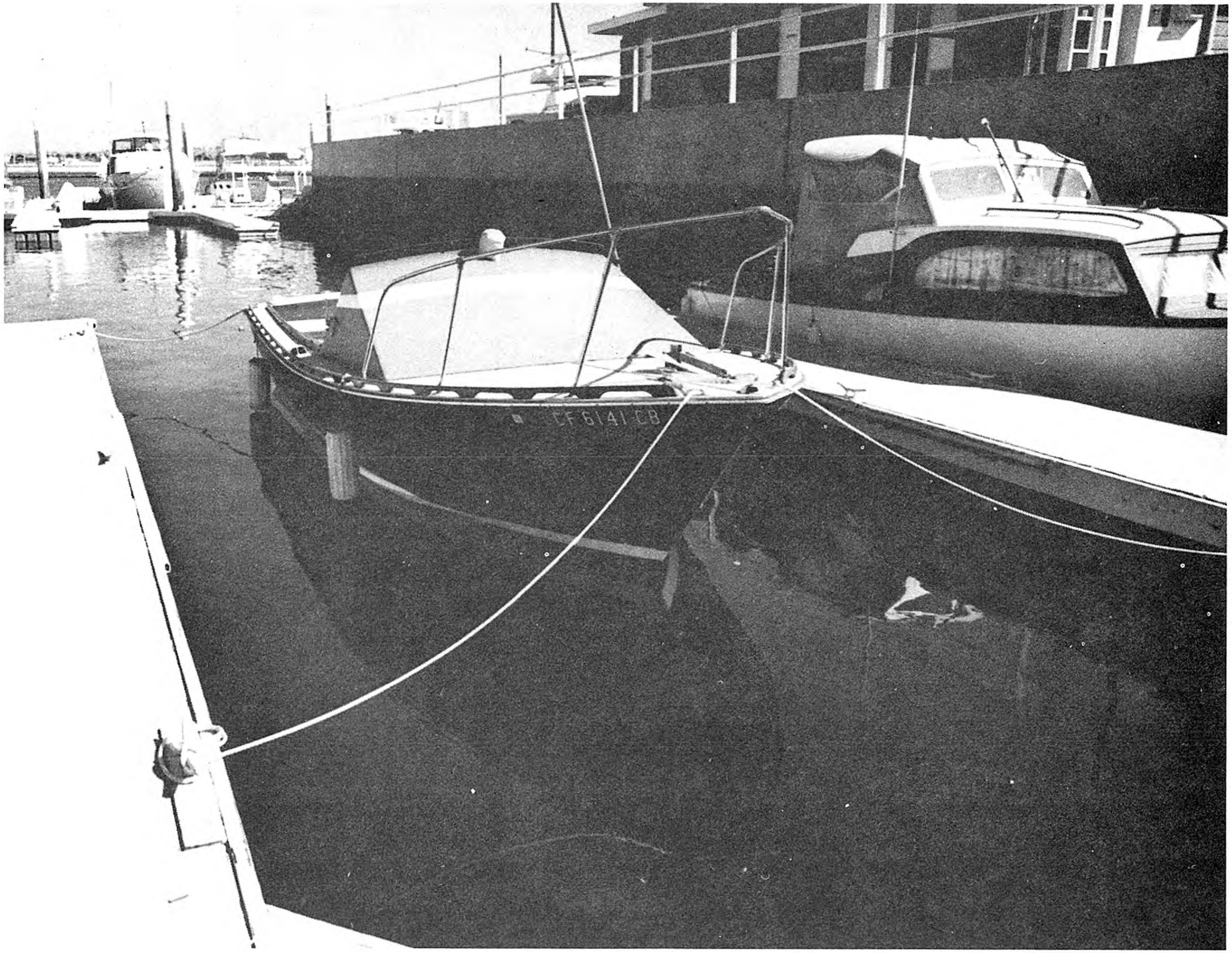

Fig. 5.3. 22 foot-5 inch Moored Boat: Reg. No. CF 6141 CB

Fig. 5.4. 24 foot-8 inch Moored Boat: Reg. No. CF 0675 CJ

Fig. 5.5. 25 foot Moored Boat: Reg. No. CF 0394 CV

Fig. 5.6. 25 foot Moored Boat: Reg. No. CF 0310 CV

Fig. 5.7. 29 foot-3 inch Moored Boat: Reg. No. CF 7651 CL

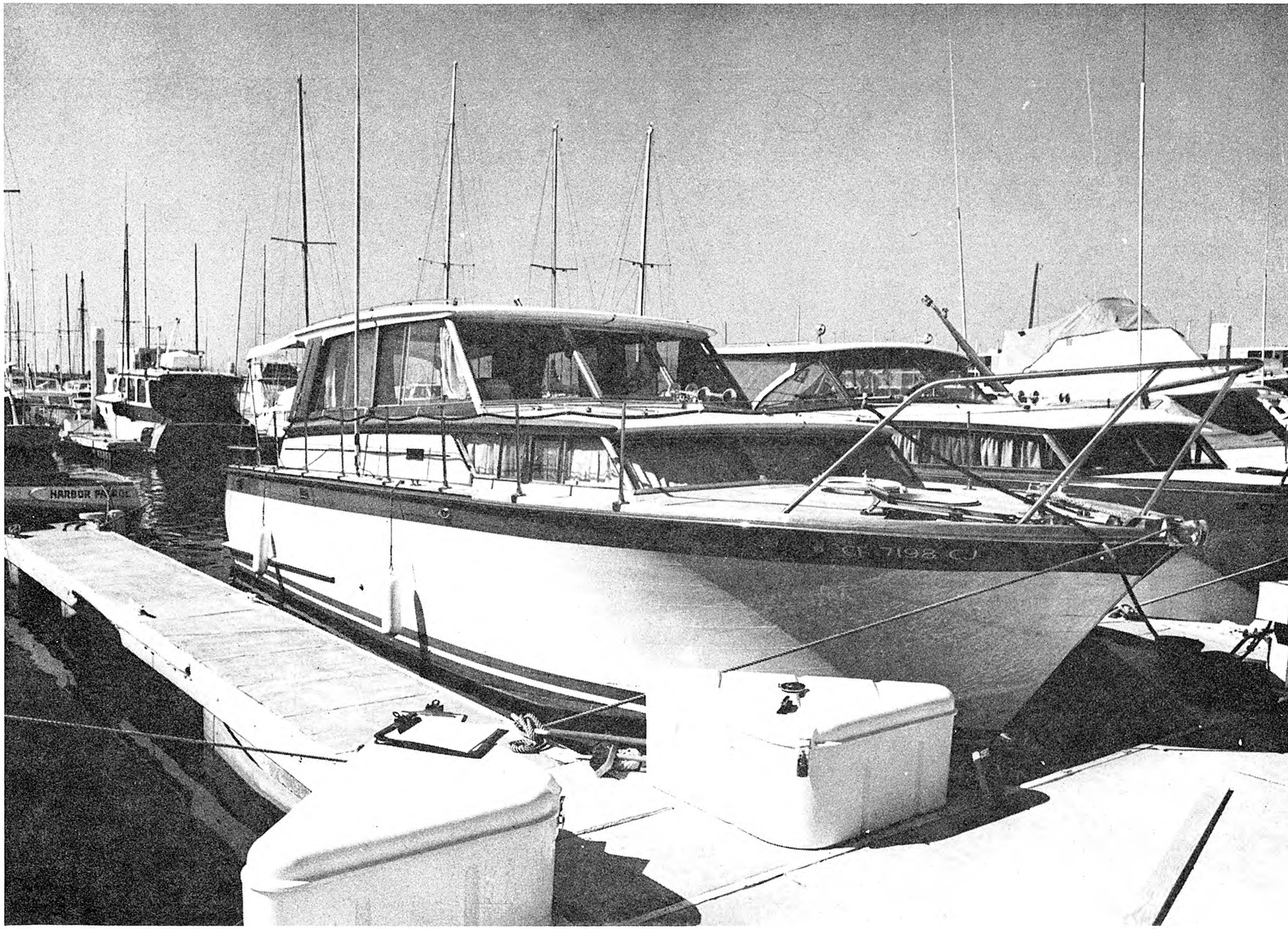

Fig. 5.8. 33 foot Moored Boat: Reg. No. CF 7198 CJ

Fig. 5.9. 38 foot Moored Boat: Reg. No. CF 1756 CW

as large a boat as is possible into a slip the clearance between boat and slip is minimized and as mentioned previously the probability of asymmetrical restoring forces is increased. With the latter effect permitting larger motions of the small moored boat in one direction than another, the impact of boat and dock when the boat moves in surge becomes an important possible source of damage.

The dimensions of the mooring system of the seven boats described in Table 5.1 are presented in Table 5.2 using the nomenclature of Fig. 2.8. For each boat and for each mooring line the mooring dimensions, material, diameter and condition of the lines are presented. It is noted that the dimension f has associated with it either a plus or a minus sign. This sign indicates the direction of the restoring force. For instance, for the mooring system shown in Fig. 5.3 (Registration No. CF 6141 CB) the distance f is positive. In that case the bow lines go from the boat to the dock in such a way that when the boat is displaced in a direction toward the stern the bow lines restrain the boat and provide the restoring force. When the motion of the boat in surge is toward the bow, the stern lines become the active restoring force; hence, the distance f is indicated as being positive for this case also. Conversely for the 24 foot-8 inch boat (Registration No. CF 0675 CJ) the bow lines become active for displacements of the boat toward the bow and the stern lines become active for displacements of the boat in the direction of the stern. Therefore, in that case the dimension f for both bow and stern lines is negative. As another example of the sign convention, for the 33 foot boat (Registration No. CF 7198 CJ) the bow

Table 5.2. Dimensions, Material, Diameter, and Condition of Existing Mooring Lines, in situ Measurements: Marina del Rey (10/17/67)

State of Calif. Registration No.	Bow, stern or spring-line	Starboard or Port	f (ft)	z_o (ft)	y_o (ft)	ℓ (ft)	$\Delta\ell$ (ft)	MOORING LINES: Material	Dia. (in.)	Condition
CF 6141 CB	bow	starboard	+ 4.08	2.0	6.83	8.2	0.0	nylon	0.5	good
	bow	port	+ 4.05	2.0	7.0	8.33	0.0	nylon	0.5	good
	stern	starboard	+ 3.21	1.08	4.33	5.49	0.5	nylon	0.5	good
	stern	port	+ 2.75	1.42	3.92	5.08	0.33	nylon	0.5	good
CF 0675 CJ	bow	starboard	- 1.09	2.50	3.98	4.82	0.0	manila	0.5	good
	bow	port	- 1.34	2.67	4.82	5.67	0.08	manila	0.5	good
	stern	starboard	- 0.09	2.17	2.11	3.21	0.38	manila	0.5	good
	stern	port	- 0.63	2.17	1.59	2.75	0.0	manila	0.5	good
CF 0394 CV	bow	starboard	- 1.46	3.13	4.93	6.03	0.0	nylon	0.5	good
	bow	port	- 1.67	3.42	3.75	5.35	0.0	nylon	0.5	good
	stern	starboard	- 0.67	2.21	2.08	3.11	0.0	nylon	0.5	good
	stern	port	- 0.75	2.17	1.13	2.55	0.0	nylon	0.5	good
CF 0310 CV	bow	starboard	- 2.29	3.17	4.88	6.25	0.08	manila	0.5	fair
	bow	port	- 2.21	3.08	3.46	5.14	0.12	manila	0.5	poor
	stern	starboard	- 0.08	2.29	1.67	2.83	0.0	nylon	0.375	good
	bow	port	- 0.46	2.46	1.42	2.87	0.0	nylon	0.375	good
	spring-line	port	+ 8.68	2.37	0.83	8.96	0.291	nylon	0.375	good

Table 5.2. Dimensions, Material, Diameter, and Condition of Existing Mooring Lines, in situ measurements: Marina del Rey (10/17/67) (cont'd)

State of Calif. Registration No.	Bow, stern or spring-line	Starboard or Port	f (ft)	z_o (ft)	y_o (ft)	ℓ (ft)	$\Delta\ell$ (ft)	MOORING LINES		
								Material	Dia. (in.)	Condition
CF 7651 CL	bow	starboard	- 5.58	3.21	6.92	9.45	0.0	nylon	0.5	good
	bow	port	- 6.25	3.21	7.25	10.10	0.0	nylon	0.5	good
	stern	starboard	*	---	---	---	---	---	---	---
	stern	port	- 1.0	2.75	4.42	5.30	0.0	nylon	0.5	good
	spring-line	starboard	+12.25	1.92	1.71	12.51	0.0	nylon	0.5	good
CF 7198 CJ	bow	starboard	- 6.00	3.33	7.42	10.10	0.0	nylon	0.625	good
	bow	port	- 5.67	3.33	6.67	9.36	0.0	nylon	0.625	good
	stern	starboard	+ 0.71	2.63	4.08	4.91	0.0	nylon	0.625	good
	stern	port	+ 0.75	2.50	1.29	2.91	0.0	nylon	0.625	good
	spring-line	port	+ 8.75	2.96	1.17	9.3	0.0	nylon	0.625	good
CF 1756 CW	bow	starboard	- 4.67	4.46	6.0	8.85	0.75	manila	0.75	good
	bow	port	- 4.75	4.65	6.67	9.43	0.417	manila	0.75	good
	stern	starboard	+ 2.10	2.79	1.34	3.73	0.0	manila	0.75	good
	stern	port	+ 2.17	2.87	1.84	4.08	0.0	manila	0.75	good
	spring-line	starboard	+ 4.92	2.83	0.92	5.75	0.17	nylon	0.625	good
	spring-line	port	+ 4.83	3.17	1.25	5.92	0.0	nylon	0.625	good

*This line too slack to be considered as contributing to restraining force.

and the stern lines provide a restoring force, for reasonably small motions, only when the direction of motion of the boat is toward the bow. For that boat, the only restoring force for motion of the boat toward the stern (negative x-direction) is provided by the spring-line. In all cases investigated, the spring-line goes from the port or starboard midsection of the boat to the dock in a direction toward the bow. Therefore, the spring-lines tend to hold the boat forward in the slip and provide a restraining force for surge motions of the boat toward the stern (negative x-direction). It is noted in Table 5.2 that the length of bowline on the boat, the quantity ℓ_1 in Fig. 2.8, is not included. In these cases this length was small compared to the line length. (For Harbor Boat No. 3 it was found that this section of the line does not significantly effect either the periods of the free oscillation of the boat or the characteristics of the forced oscillations.)

5.3 Predicted Restoring Force

In order to determine the variation of the restoring force with displacement in the x-direction, the dimensions of Table 5.2 and the constants presented in Tables 3.1 and 3.2 were used in conjunction with Eq. 2.58 to first determine the variation of the component of the tension of the line in the x-direction, T^*_x, with displacement. For a given displacement (in either the positive x-direction or the negative x-direction) the values of T^*_x for all active lines were then summed in accordance with Eq. 2.46. The resultant curves are presented in Figs. 5.10 through 5.16 for the seven boats whose dimensions and mooring system characteristics are described in Tables 5.1 and 5.2.

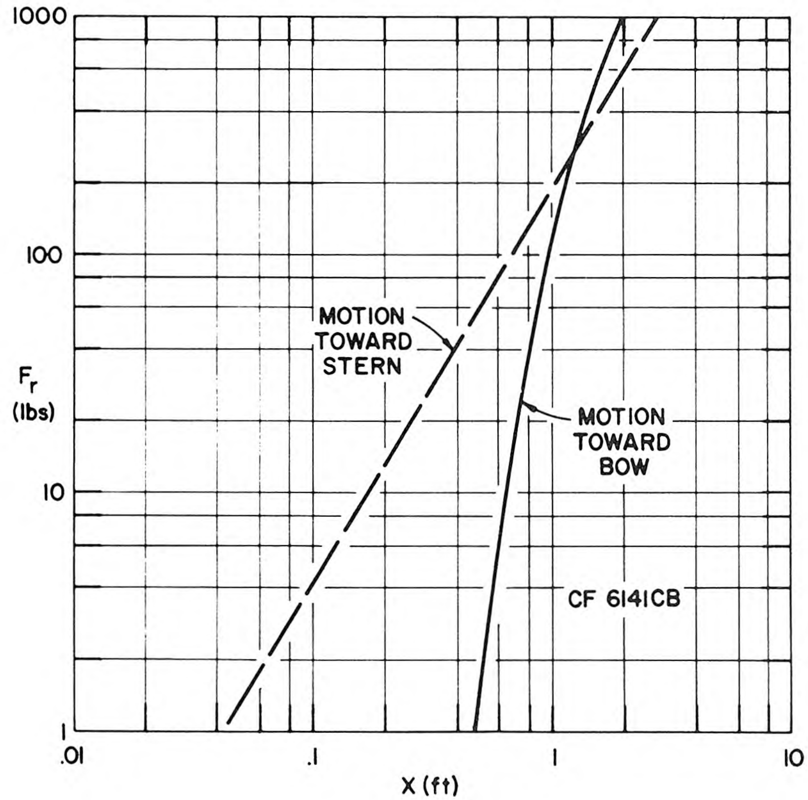

Fig. 5.10. Predicted Restoring Force vs. Displacement of 22 foot-5 inch Moored Boat: Reg. No. CF 6141 CB

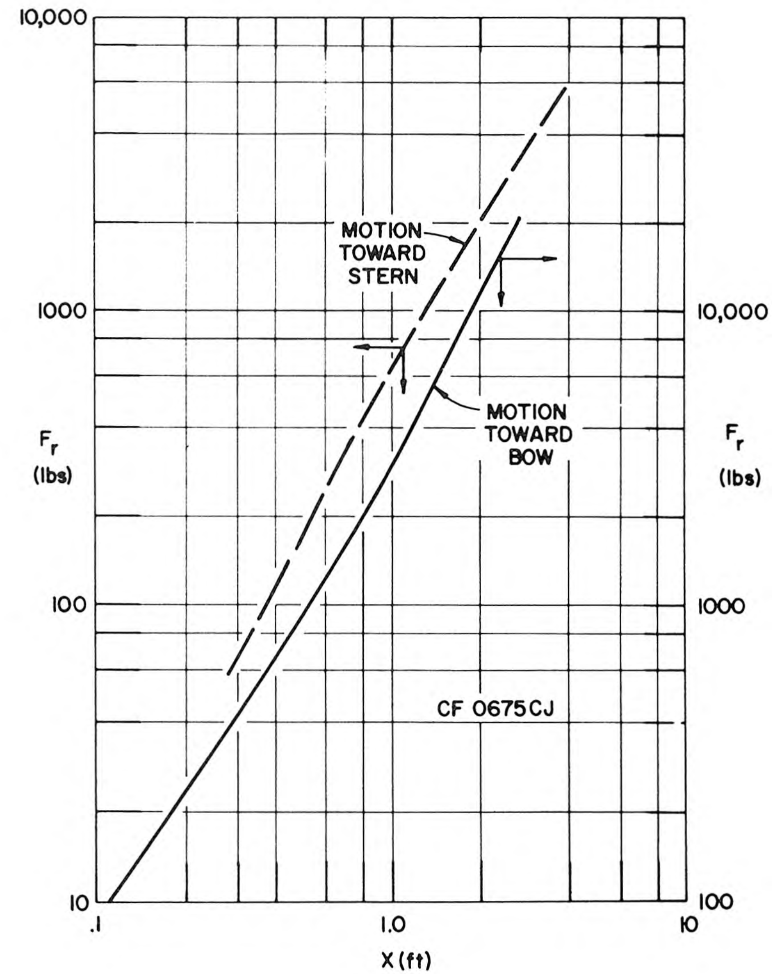

Fig. 5.11. Predicted Restoring Force vs. Displacement of 24 foot-8 inch Moored Boat: Reg. No. CF 0675 CJ

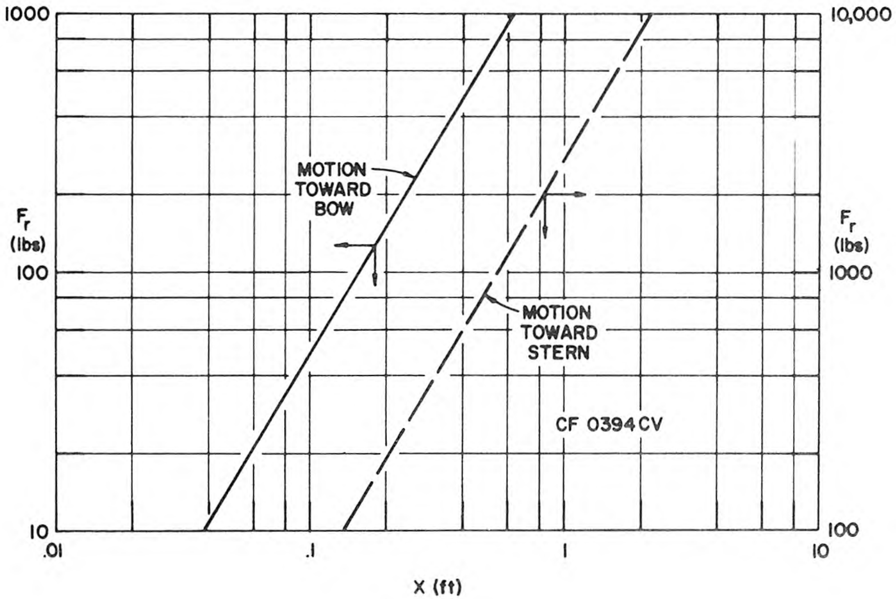

Fig. 5.12. Predicted Restoring Force vs. Displacement of 25 foot Moored Boat: Reg. No. CF 0394 CV

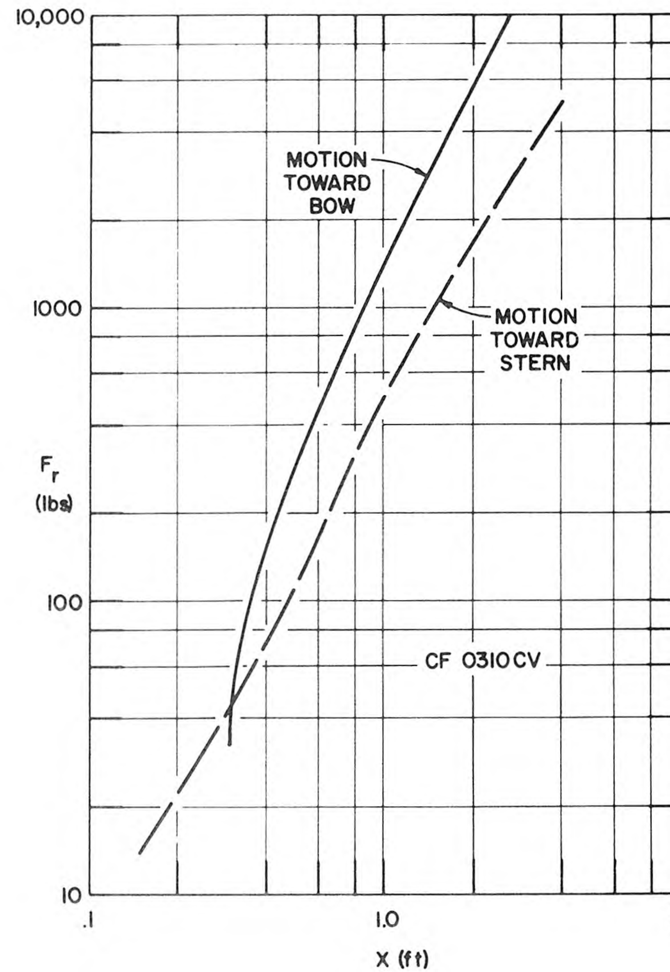

Fig. 5.13. Predicted Restoring Force vs. Displacement of 25 foot Moored Boat: Reg. No. CF 0310 CV

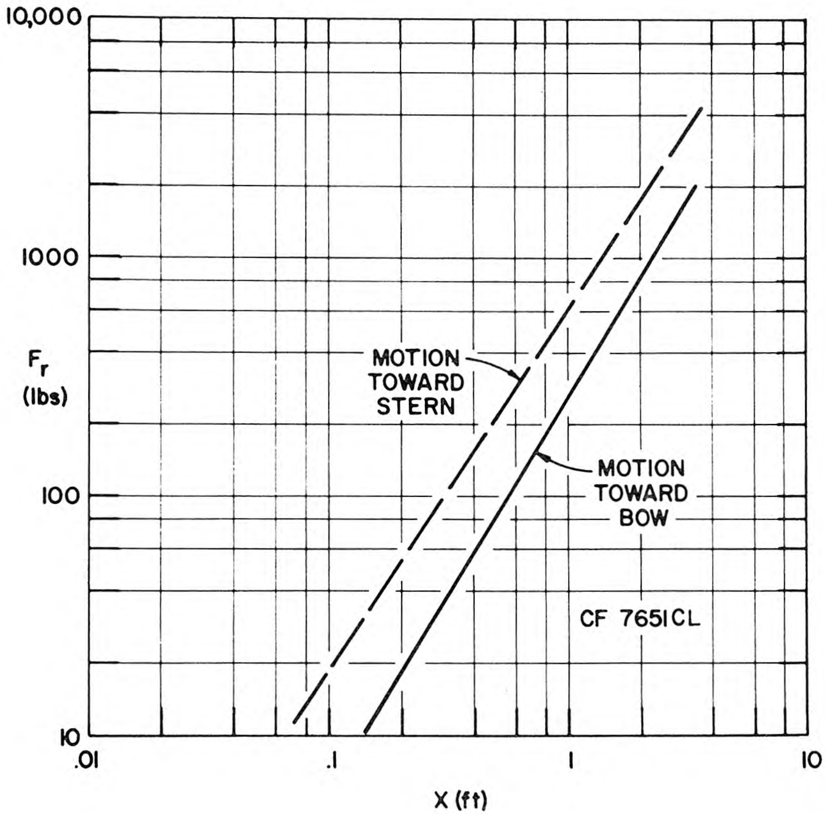

Fig. 5.14. Predicted Restoring Force vs. Displacement of 29 foot-3 inch Moored Boat: Reg. No. CF 7651 CL

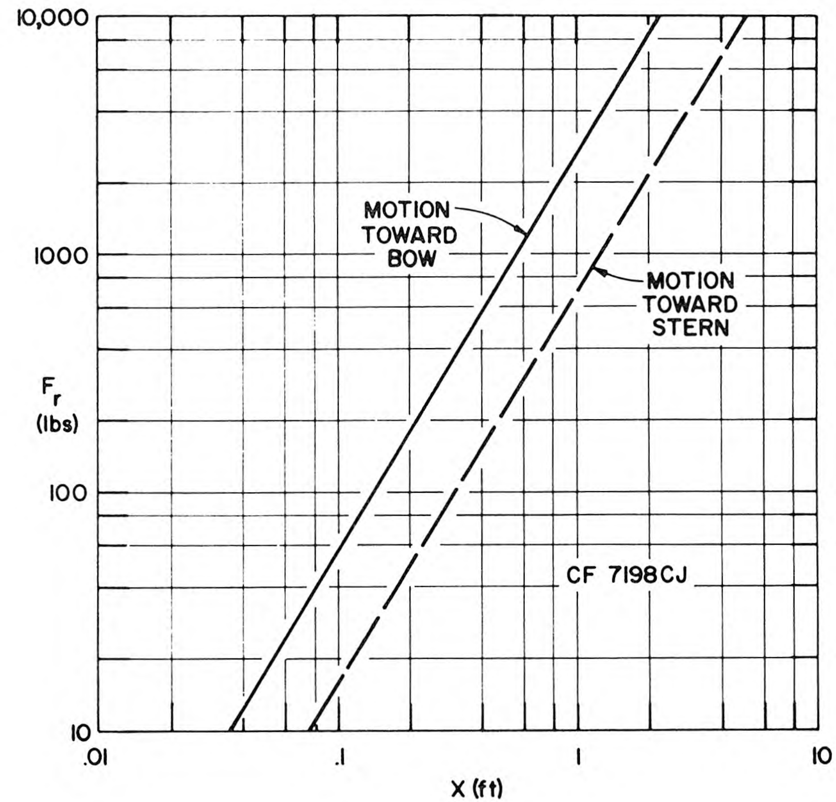

Fig. 5.15. Predicted Restoring Force vs. Displacement of 33 foot Moored Boat: Reg. No. CF 7198 CJ

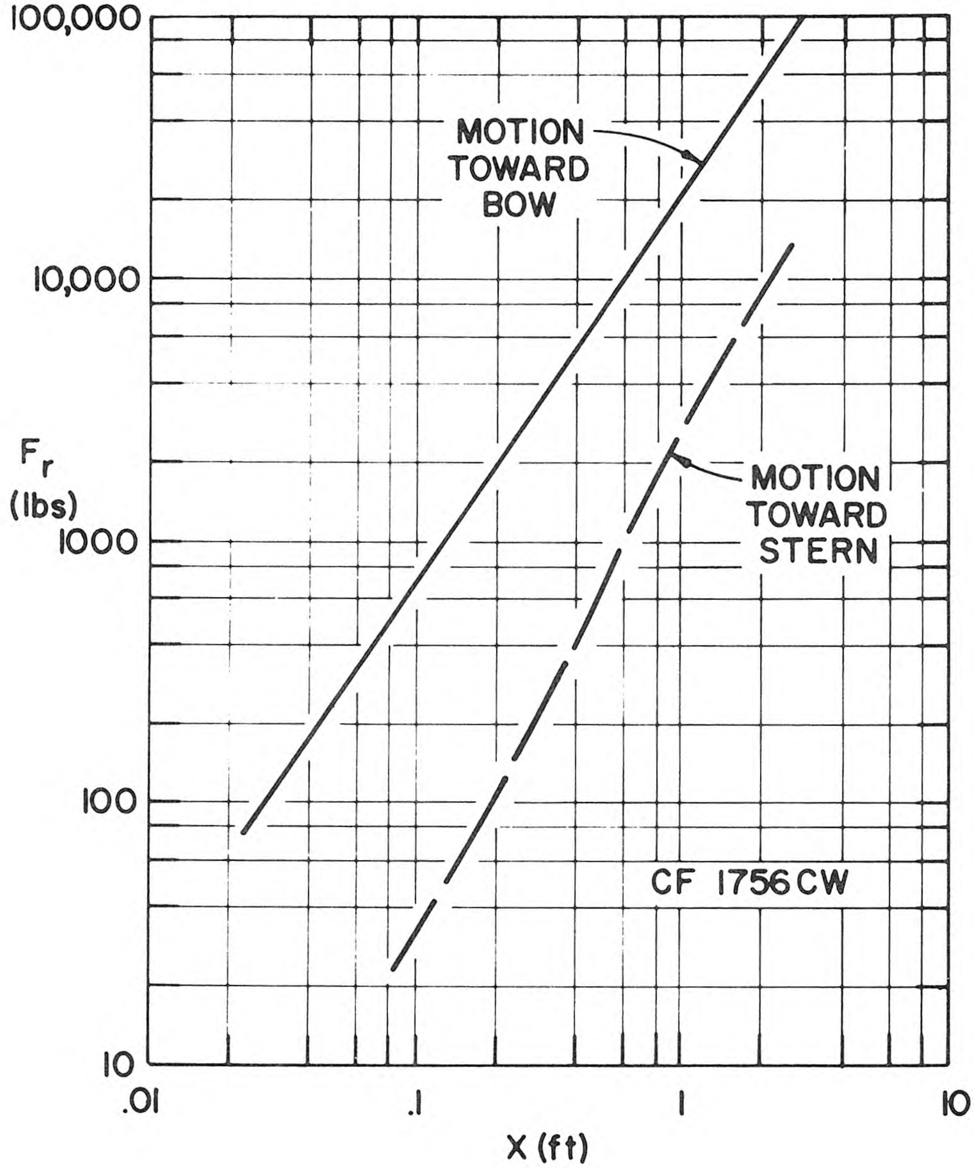

Fig. 5.16. Predicted Restoring Force vs. Displacement of 38 foot Moored Boat: Reg. No. CF 1756 CW

Consider first Figs. 5.12, 5.14, and 5.15 which are for boats moored with all lines taut ($\Delta \ell = 0$). For these boats the free travel, Δ_f, is equal to zero and the x-component of the tension, T^*_x, is directly proportional to a power of the displacement (the power given by Table 3.1 for the appropriate material). The remaining figures, Figs. 5.10, 5.11, 5.13, and 5.16,indicate a certain amount of free travel, since as the applied force decreases the displacement approaches a constant value. The exact amount of the free travel is difficult to evaluate due to the inherent difficulty in determining the amount of slack in the mooring lines. For instance, Fig. 5.10 indicates that the bow lines are taut while the stern lines are slack. However, if the at-rest-position of the boat could be accurately determined, both the bow and the stern lines would have some slack in them. Therefore, the restoring force curves of boats moored with slack lines must be viewed with some judgment, since although the total free travel from the furthest forward to the furthest position rearward is correct, as before, the location of the at-rest-position of the boat may be questionable.

The curves shown in Figs. 5.10 through 5.16 were fitted with the expressions described by Eq. 2.33. It was necessary to determine the coefficients in these equations so that the response of the moored boats could be predicted in accordance with the method presented in Section 2. The values of the coefficients of Eq. 2.33 are presented in Table 5.3 and indicate one of the limitations of this type of analysis. Consider the values of the coefficients r_3. For all the cases considered this coefficient was negative. Since this is the coefficient of the fifth order term of $F_2(x)$, the maximum value of the displacement to be

Table 5.3. Coefficients Used in Fitting Eqs. 2.33 to Restoring Force Curves

State of California Registration No.	x > 0			x < 0		
	a_1 ($lb\text{-}ft^{-1}$)	a_2 ($lb\text{-}ft^{-3}$)	a_3 ($lb\text{-}ft^{-5}$)	r_1 ($lb\text{-}ft^{-1}$)	r_2 ($lb\text{-}ft^{-3}$)	r_3 ($lb\text{-}ft^{-5}$)
CF 6141 CB	- 28.27	125.48	1.65	39.39	160.98	- 18.37
CF 0675 CJ	178.76	460.92	- 29.68	1201.81	1844.95	-146.76
CF 0394 CV	161.20	99.40	- 6.38	434.0	2850.0	-532.0
CF 0310 CV	- 71.72	2010.0	- 310.52	91.04	539.98	- 89.44
CF 7651 CL	68.89	239.40	- 27.98	180.36	464.52	- 54.88
CF 7198 CJ	534.85	2518.10	- 299.85	152.97	503.23	- 53.51
CF 1756 CW	6620.0	17400.0	-2130.0	292.0	2780.0	-320.0

investigated when determining the response curves must not exceed the maximum value of the displacement used in developing the restoring force curves. If this limit is exceeded this term will dominate and the fitted expression will deviate significantly from the predicted curve. This is not considered a serious limitation, since the objective of the analysis is to predict the range of wave periods of importance for moored small boats and not necessarily to predict the exact value of the wave period at which lines will part. Of course, this latter period could be determined simply by first evaluating the restoring forces up to the point at which the lines break ($T^* = T^*_{brk}$) and then fitting the appropriate expression (either $F_1(x)$ or $F_2(x)$) to the resulting curves.

Some of the asymmetrical aspects of this non-linear mooring problem can be appreciated from a comparison of the order of magnitude of the different coefficients a and r presented in Table 5.3. For instance, the largest boat studied (Registration No. CF 1756 CW) has a much stiffer mooring system opposing motion in the positive x-direction (displacement from the at-rest-position in a direction toward the bow) than surge motions toward the stern (negative x-direction). Conversely the boat with Registration No. CF 7651 CL has similar restoring forces for both directions of boat movement. As described in Section 2.1.2 and particularly in Fig. 2.4 the degree of asymmetry of mooring is quite well described by the variation of θ_1 with the maximum motion X_{max}. The quantity θ_1 was defined as the value of σt when the boat passes through the at-rest-position. As described previously, when θ_1 is equal to $\pi/2$ the restoring system is considered to be symmetrical, when θ_1 is equal to zero or π the motion is highly asymmetrical.

Fig. 5.17 shows the variation of θ_1 with the maximum displacement X_{max} for the seven boats investigated in this study. The value of θ_1 was obtained by solving Eq. 2.36 using the coefficients presented in Table 5.3. For these cases, four boats have values of θ_1 greater than $\pi/2$ and three boats have values of θ_1 less than $\pi/2$ for the displacements shown. Referring to the definition sketch, Fig. 2.4, this indicates that the four boats with $\theta_1 > \pi/2$ have stiffer mooring systems resisting motions for $x < 0$ than for $x > 0$. The reverse is true for the three boats with $\theta_1 < \pi/2$. This variation is confirmed by the values of the coefficients presented in Table 5.3.

The following numerical example is used to further demonstrate the application of the data presented in Fig. 5.17 to the problem of evaluating the degree of asymmetry of a moored boat. Consider the boat with Registration Number CF 0675 CJ and a maximum displacement of 1 foot. From Fig. 5.17 the value of θ_1 is 1.9 radians (or 109°) and the maximum displacement is given by $X_{max} = X(1 - \cos \theta_1)$ from the definitions which accompany Eq. 2.38. Therefore: $X_{max} = 1.33\ X$ and in a similar way $X_{min} = 0.67X$, or for the conditions which are considered the displacement of the boat from the at-rest-position in the direction of the bow is twice the displacement in the direction of the stern.

It is interesting to postulate as to the application of this method to the case of large ships. One important problem for large ships is the sway motion of the vessel, i.e., motion perpendicular to the dock.

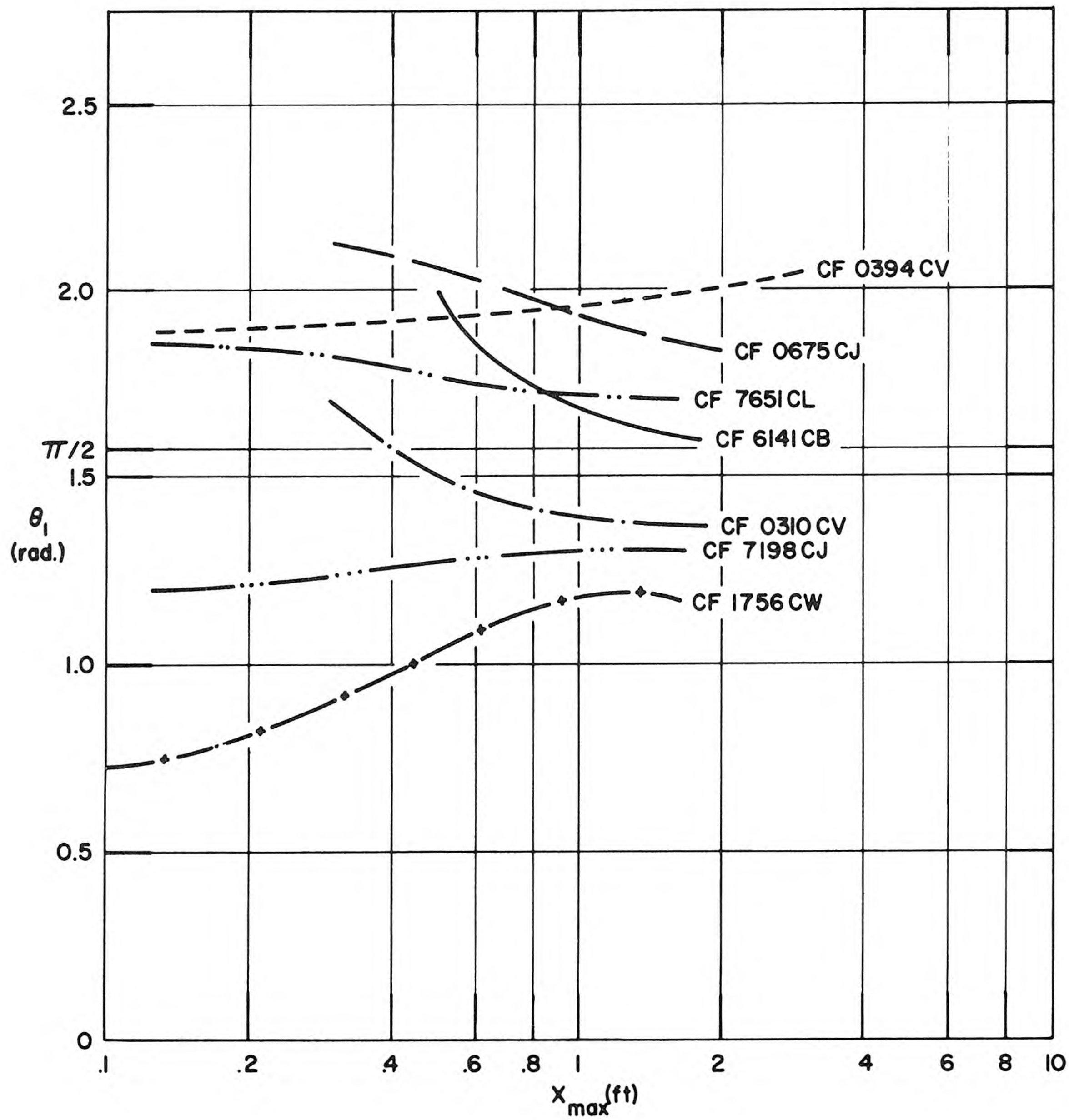

Fig. 5.17. Variation of θ_1 with X_{max}

In that case elastic lines usually restrict motion away from the dock and a very stiff fender system restricts motion in a direction toward the dock. Considering these motions the restoring forces (and the resulting coefficients of the fitted equations) would be much greater for sway motions when $y < 0$ than when $y > 0$. Hence θ_1 would approach π for this case, and the motion would be highly asymmetrical. Therefore, the motion away from the dock would be approximately twice what one would expect if the boat had been assumed to be symmetrically moored with elastic lines.

5.4 Predicted Response Curves

Response curves which describe the dynamics of the motion of the seven small boats in surge are presented in Figs. 5.18 through 5.24. These curves are shown as the variation of maximum motion in the positive x-direction, X_{max}, and the maximum motion in the negative x-direction, X_{min}, with wave period T for constant values of the forcing function ζ. The role of the parameter ζ has been fully discussed in Section 4.5 and will not be discussed here. Nevertheless, it should again be emphasized that the ratio ζ/A is also a function of wave period and in order to fully determine the forced oscillations of a small boat moored in a standing wave environment, the variation of ζ/A with wave period must also be evaluated. However, nearly irrespective of the value of ζ for a particular wave period, the response curves presented in terms of constant values of ζ indicate the important range of wave periods with respect to surge motions for the small boats which were considered.

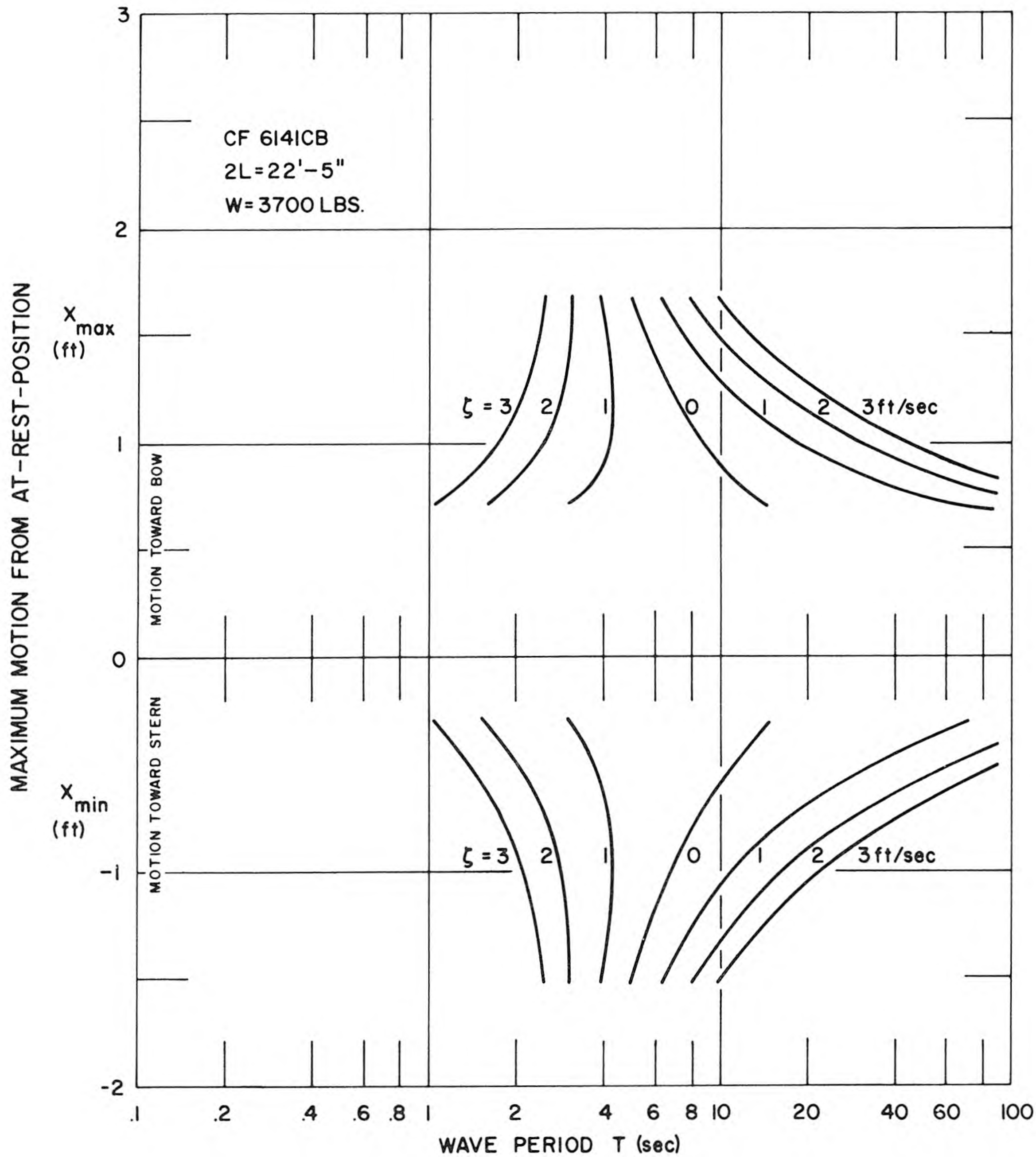

Fig. 5.18. Response Curve, 22 foot-5 inch Moored Boat: Reg. No. CF 6141 CB

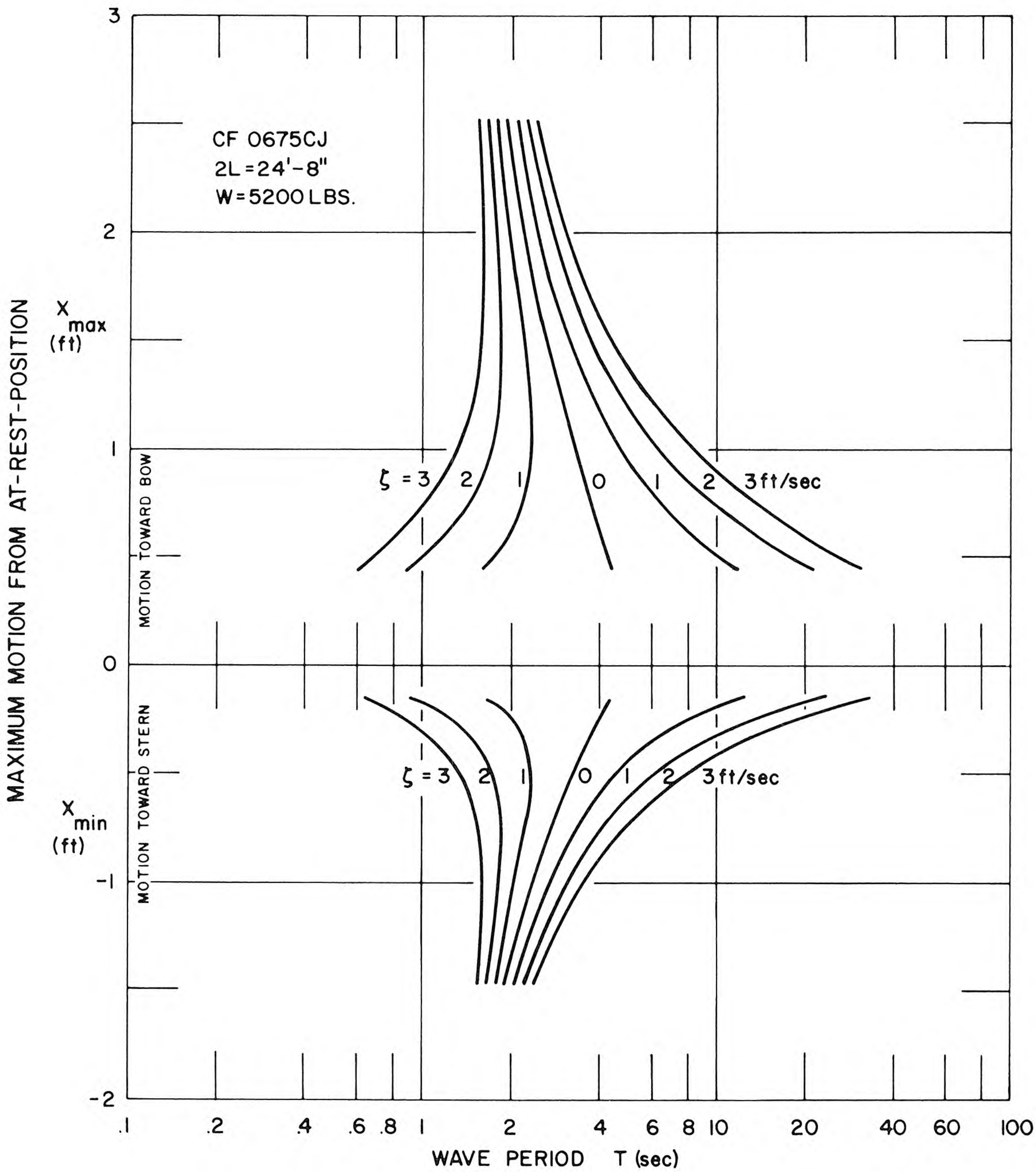

Fig. 5.19. Response Curve, 24 foot-8 inch Moored Boat: Reg. No. CF 0675 CJ

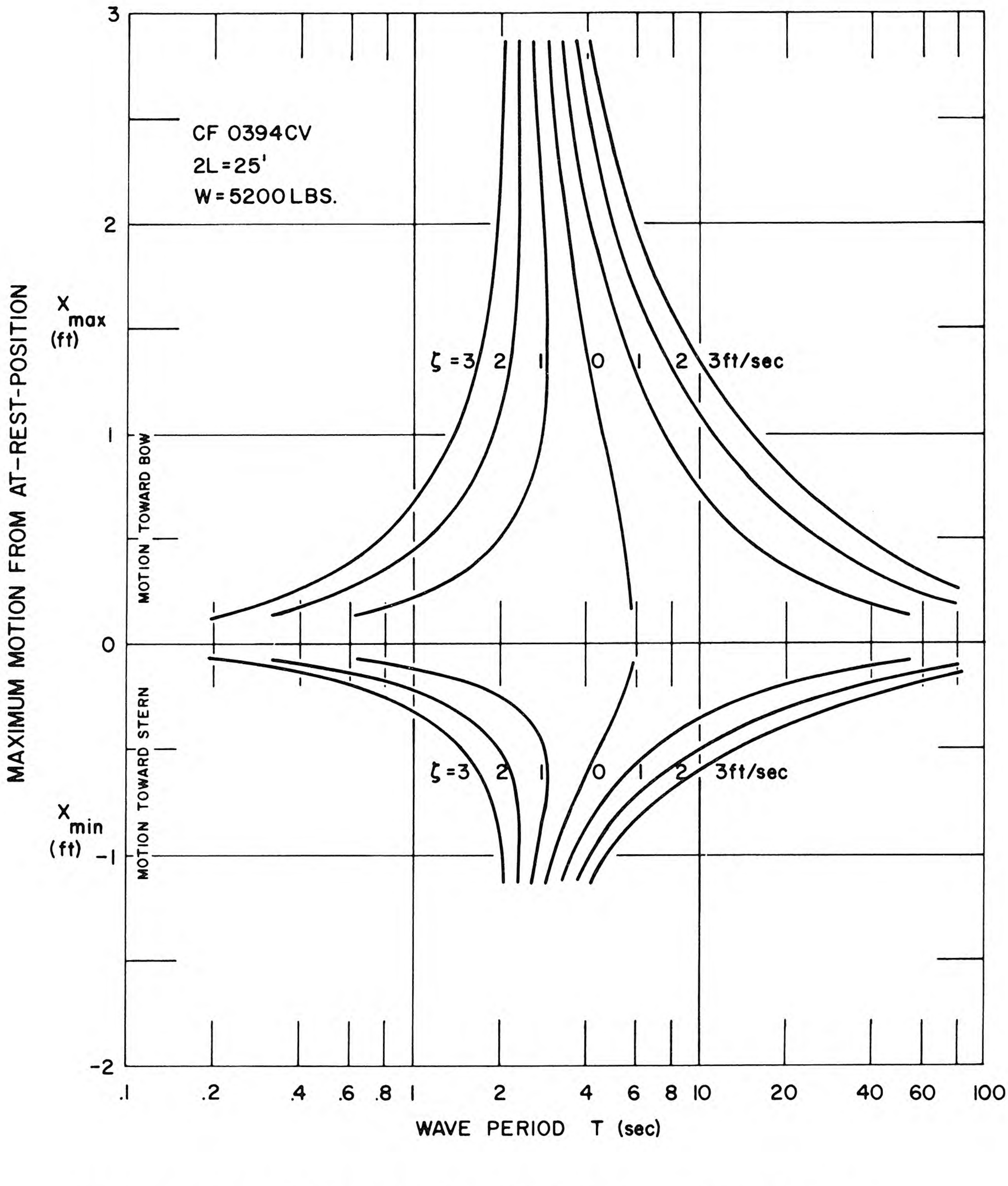

Fig. 5.20. Response Curve, 25 foot-Moored Boat: Reg. No. CF 0394 CV

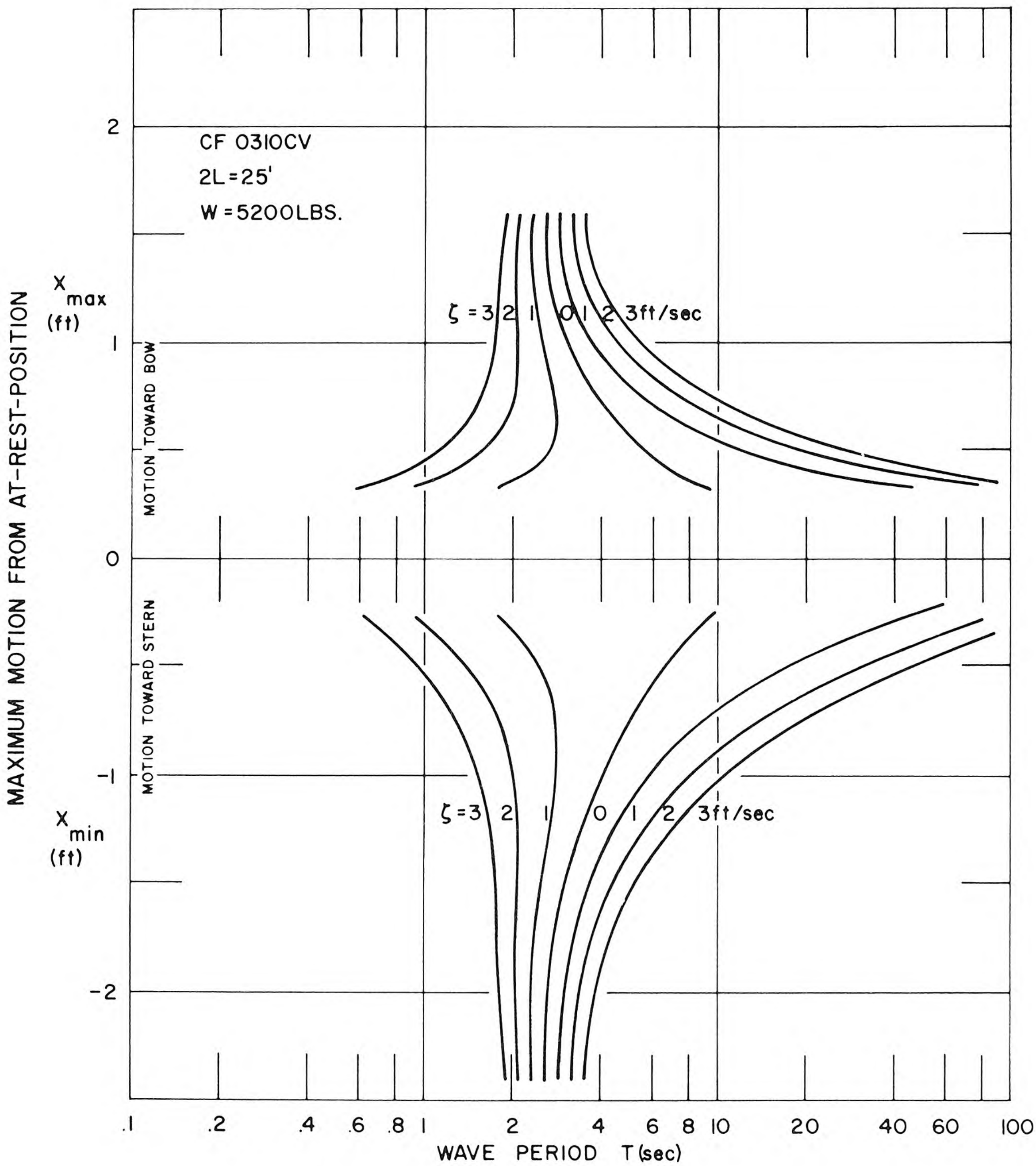

Fig. 5.21. Response Curve, 25 foot Moored Boat: Reg. No. CF 0310 CV

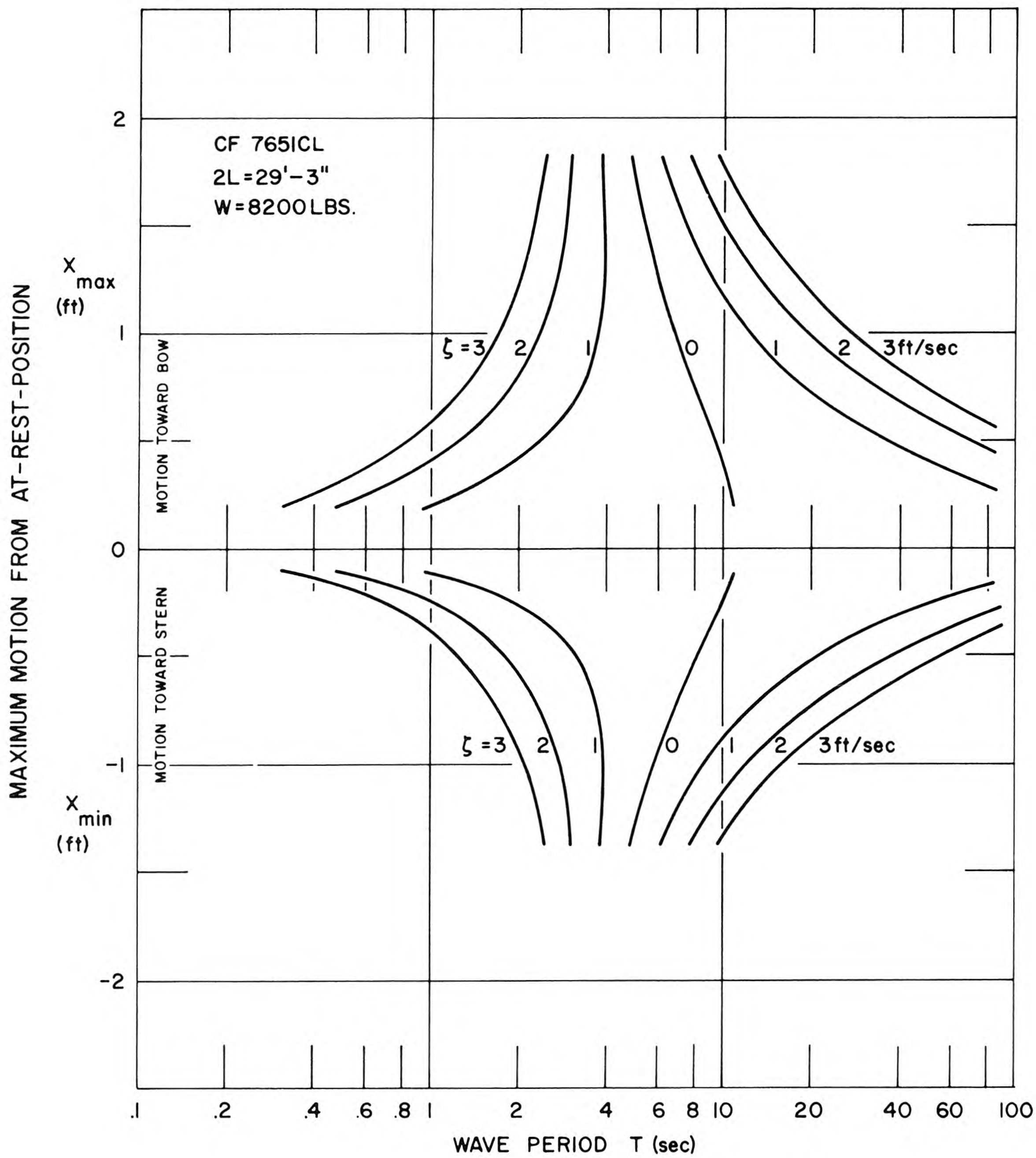

Fig. 5.22. Response Curve, 29 foot-3 inch Moored Boat: Reg. No. CF 7651 CL

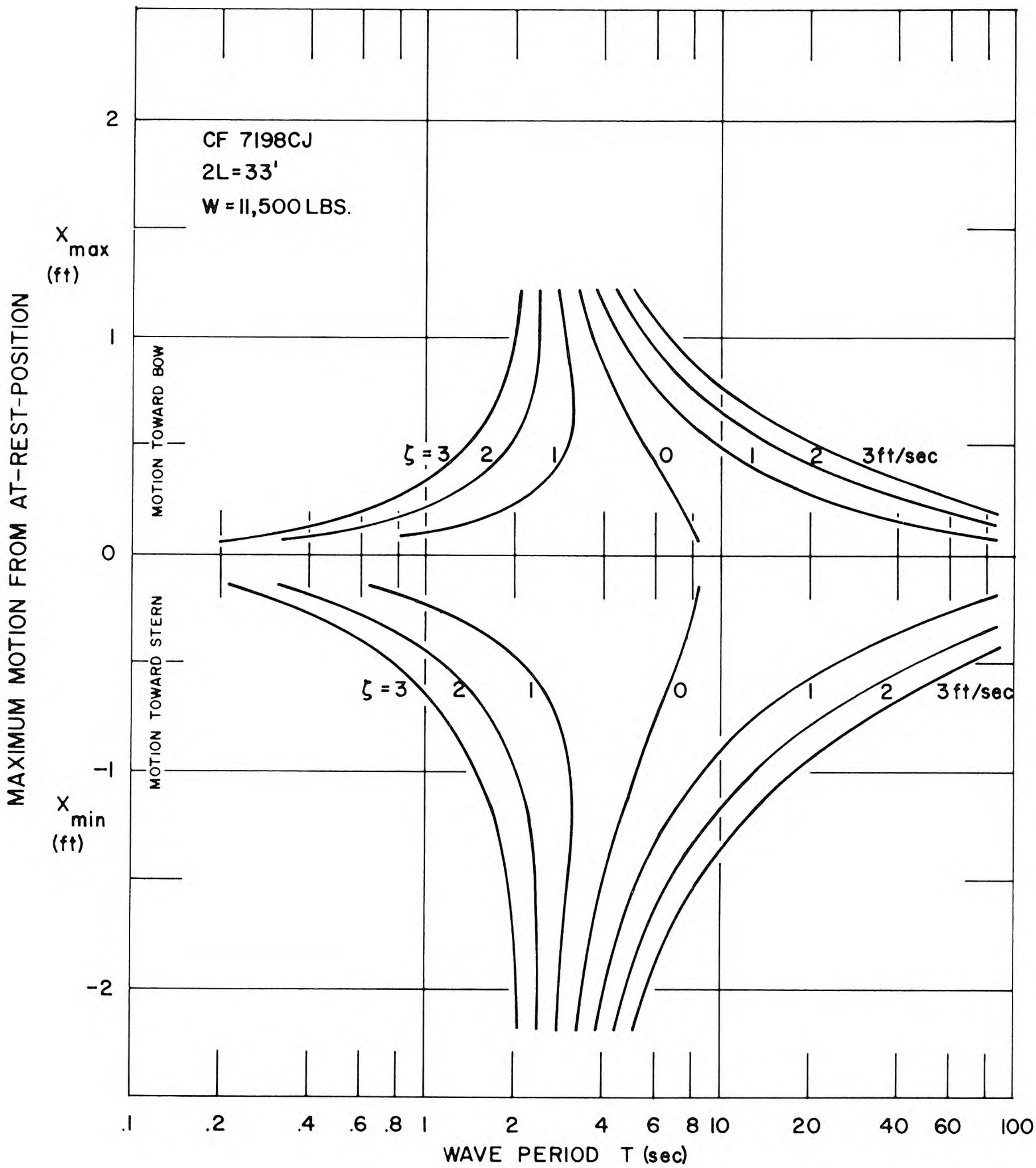

Fig. 5.23. Response Curve, 33 foot Moored Boat: Reg. No. CF 7198 CJ

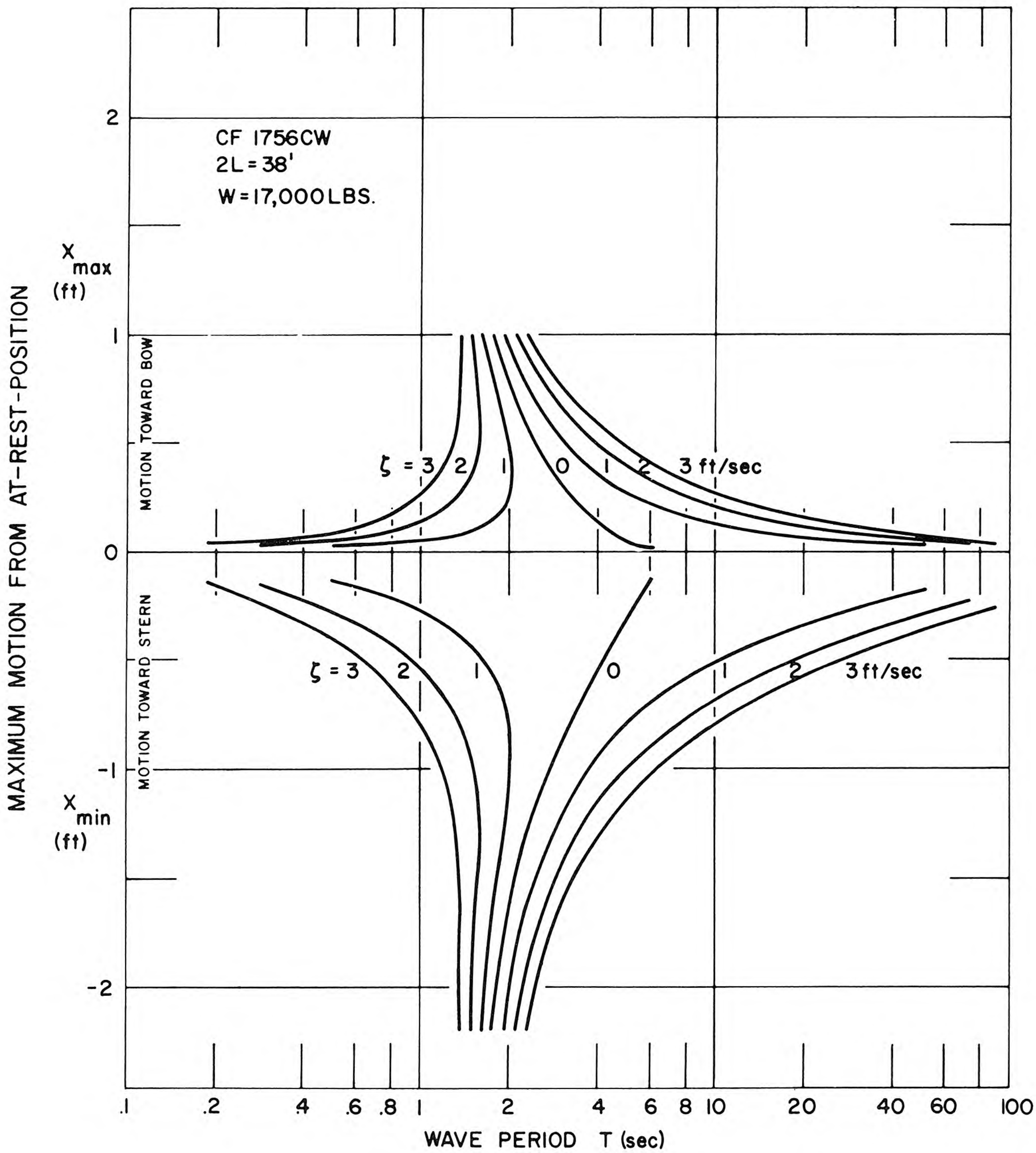

Fig. 5.24. Response Curve, 38 foot Moored Boat: Reg. No. CF 1756 CW

The response curves have been determined from Eq. 2.38 and the other necessary system parameters in a similar fashion as the response curves for Harbor Boat No. 3 were obtained. A virtual mass coefficient (C_M in Eq. 2.38) of 1.34 was used in these calculations for all seven boats. This is in accordance with the information presented by Wilson (1958) for block bodies with beam to length ratios approximately equal to 0.35. Although it is felt that improvements could be made in these results by accurately determining the virtual mass coefficients for bodies having underwater shapes closer to the small boats than that of a block body, it was not within the scope of this study to do so. It was considered important here to delineate the approximate range of important wave periods choosing a virtual mass coefficient which was somewhat conservative. (Conservative in this sense is defined as yielding ranges of resonant periods which if not exactly correct, considering other assumptions, are at least in the direction of safety, i.e., periods somewhat greater than the true values.)

There are a number of general features of these curves which warrant discussion before proceeding with a detailed discussion of certain particular characteristics. For the cases where the mooring lines are initially slack, the response curves are obtained for values of X_{max} and X_{min} which are greater than the free travel, since the response of a boat is not defined for displacements less than the free travel. In that region no restoring force is acting on the boat. For boats moored with taut lines, the "backbone" curves (curves defining the free oscillation of the small boats) are in error for small values of the boat displacement, since (as mentioned previously) the approximations which are used to describe the restoring force (Eq. 2.33) become linear with x for small x. In other words, for small displacements, due to these approximations, the non-linear aspects of the mooring disappear and the boat appears to be moored in a linear fashion; hence, the period approaches a constant value.

Fig. 5.18 shows the response of the smallest boat of the sample group ($W \cong 3700$ lbs) for several constant values of the parameter ζ. It is interesting to compare this to Fig. 5.24 which shows the response for the largest boat of the group ($W \cong 17,000$ lbs). Consider maximum displacements of 1 foot for both cases. The period of free oscillation for the small boat (Fig. 5.18) is approximately 9 sec for that case; however, for the larger boat the corresponding period is less than 2 sec. At first glance it is quite surprising that an increase in the weight of the boat by a factor of four results in a five-fold decrease in the period of free oscillation. This is due to differences in the restoring forces which can be seen by comparing Figs. 5.10 and 5.16. One obvious way of decreasing the periods of free oscillation of the smaller boat (Registration No. CF 6141 CB) would be to moor it in a smaller slip with stiffer lines, i.e., shorter lines composed of a material which is less elastic such as manila.

Some of the asymmetrical features of these non-linear moorings are shown in Fig. 5.19 (boat Registration No. CF 0675 CJ). Consider the case of a wave period of 5 sec and a forcing function $\zeta = 2$ fps. Fig. 5.19 shows that this boat would surge forward a maximum distance of approximately 1.2 feet and move in the negative x-direction a maximum of 0.6 feet for the same conditions. This would be interpreted as meaning that the mooring system which restrains motion in the direction of the stern is much stiffer than that which restrains motion toward the bow. The dimensions shown in Table 5.2 indicate that this is a case where the bow lines restrain the boat in motions toward the bow and the stern lines restrain the boat in motions toward the stern. Fig. 5.11 and Table 5.3 show that the latter lines provide much greater restraint than the former; hence, greater motions would be expected in the positive x-direction than in the negative x-direction. In this type of mooring arrangement where the boat is surrounded on three sides by the "U" shaped slip it is possible that the bow of the boat could strike the slip under the action of relatively small waves due to the asymmetrical nature of the mooring. On the other hand, if it had been moored in a larger slip this particular problem could be avoided. A positive feature of the mooring system used for this boat (Registration No. CF 0675 CJ) is that significant amplification of motions is realized for small wave periods, periods which are perhaps less than the important wave periods for most small craft harbors.

Figs. 5.19, 5.20, and 5.21 are interesting when compared as a group since they are the response curves for three boats with approximately the same shape, dimensions, and weight (25 feet long, 5200 lbs displaced weight). Response curves for the boats for a given value of ζ are different for each boat especially for the maximum motions in the negative x-direction. This emphasizes how the particular mooring system affects the dynamic response of a boat. Of the three, two of the boats were moored with manila bow lines (CF 0675 CF and CF 0310 CV, Figs. 5.19 and 5.21 respectively) and one (CF 0394 CV, Fig. 5.20) was moored with nylon bow lines. In the last case the response curves have been shifted to larger periods by the more elastic material. For motions in the positive x-direction the two boats moored with manila lines of the same diameter have similar response curves. For motions toward the stern, the response curves which are presented in Fig. 5.21 (CF 0310 CV) indicate much greater motions for the same wave period than that shown in Fig. 5.19 (CF 0675 CJ). This is reasonable, since the former boat was moored with stern lines consisting of 3/8-inch nylon compared to the latter where the stern lines consisted of 1/2-inch manila. Therefore, the reasons for the variation in the response curves among three similar boats are easily explained, and furthermore, such differences are to be expected in an ordinary small craft harbor in the absence of mooring standards.

Figs. 5.22 and 5.23 show the response curves for two boats moored with nylon lines (CF 7651 CW and CF 7198 CJ respectively). In both cases for certain boat motions the response curves are shifted to a wave period range near that of storm wave systems. However, with the stiffer line system the periods of the maximum response of the heavier boat (11,500 lbs) are smaller than those corresponding to the lighter boat (8,200 lbs), again emphasizing the importance of the mooring system on the response.

A consequence of neglecting the asymmetric nature of the mooring systems for small boats is illustrated by the response curve of one particular boat: the 25 foot, 5200 lb boat with Registration No.

CF 0394 CV. The variation of the restoring force with displacement for this case has been presented previously (Fig. 5.12) and is presented again in Fig. 5.25 so that another method of analysis, sometimes used for large ship problems, can be compared to the results of this study. The solid curve shown in Fig. 5.25 corresponds to the restoring force predicted for this case using the method of Section 2 and the appropriate elastic characteristics of the lines. The short-dashed curve in Fig. 5.25 was obtained by fitting Eq. 2.33 to the predicted restoring force using the coefficients which are presented in Table 5.3.

The response curve for ζ = 2 ft/sec has been reproduced from Fig. 5.20 in Fig. 5.26a along with the "backbone" curve for $\zeta = 0$. It is noted that this case is highly asymmetrical, due to greater restoring forces for motions in the negative x-direction than those arising for similar motions in the positive x-direction. Included in Fig. 5.26a is the variation of δ (the distance from the average position to the at-rest-position) with wave period. This shows that the motion is not symmetrical about the at-rest-position, but it is symmetrical about the mean position a distance δ away.

Consider the results of an analysis which treats the mooring system as symmetrical. Referring to Fig. 5.25, symmetrical restoring force curves are shown as long-dashed lines. These curves were determined by first plotting the predicted curves in the positive F_r vs. x quadrant and then computing the average displacement for a given applied force. The expression which best describes the resulting average curve is:

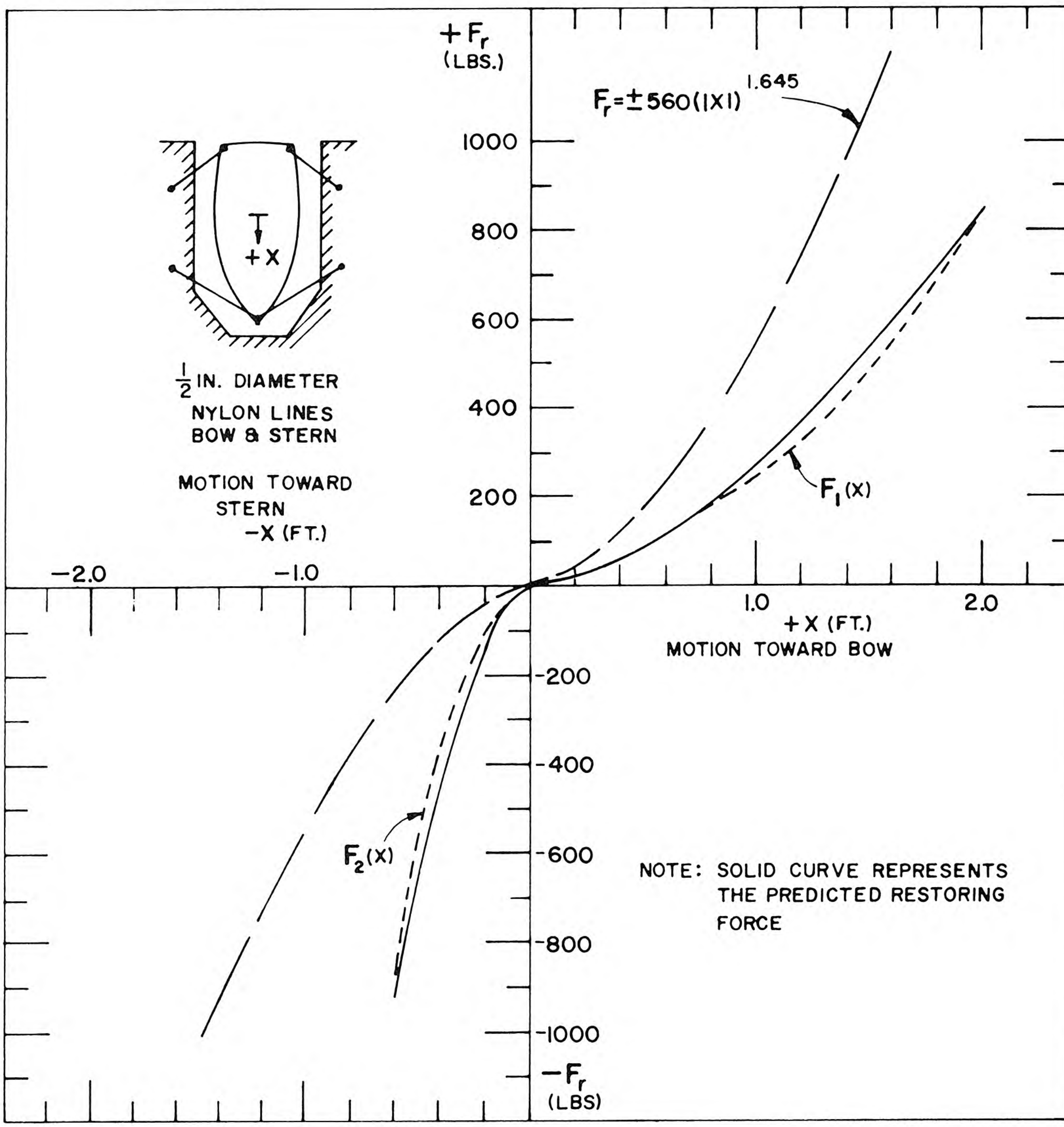

Fig. 5.25. Predicted Restoring Force vs. Displacement,
25 foot Moored Boat: Reg. No. CF 0394 CV

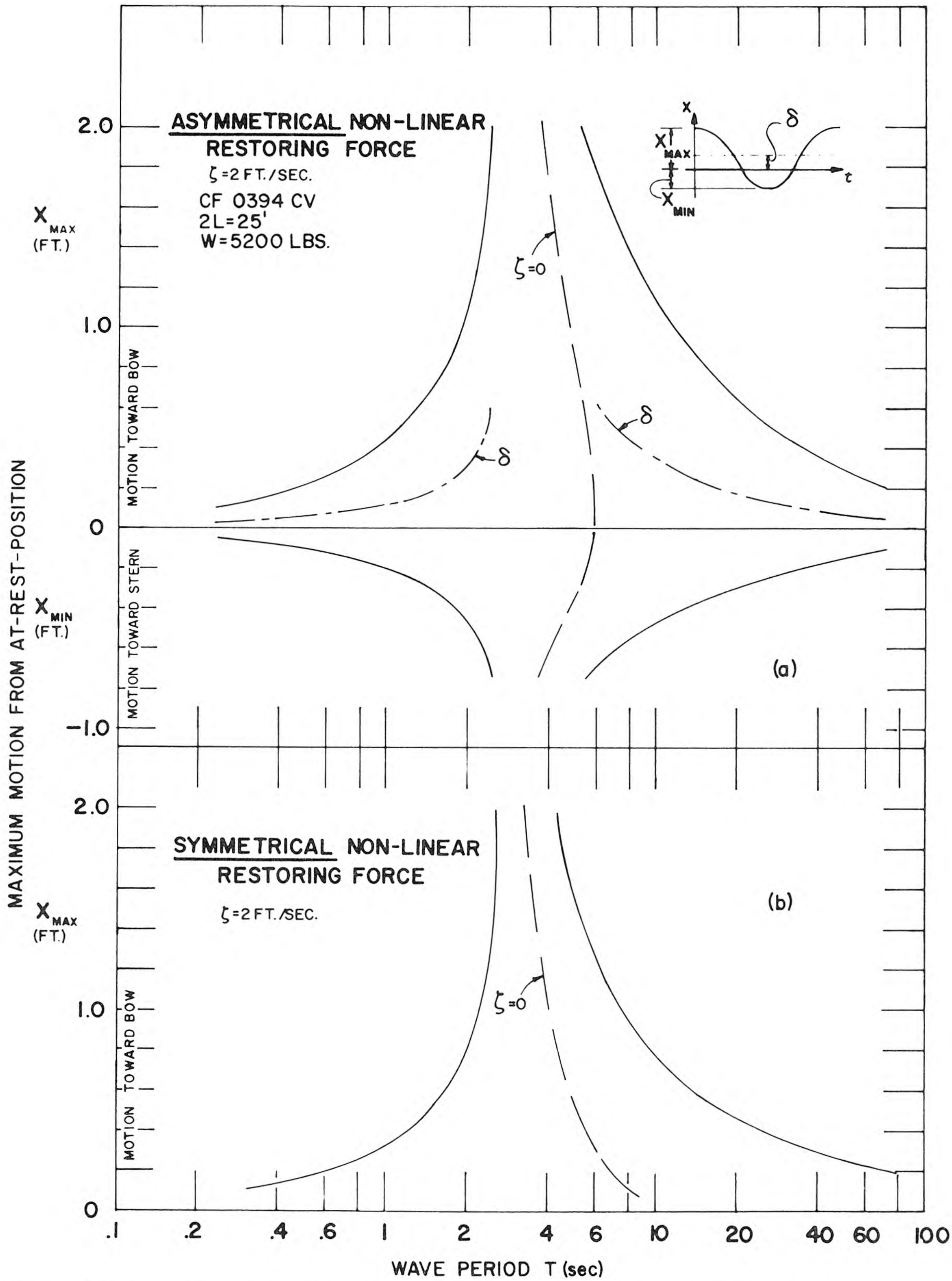

Fig. 5.26. (a) Response Curve, 25 foot Moored Boat: Reg. No. CF 0394 CV with Predicted Restoring Force

(b) Response Curve, 25 foot Moored Boat: Reg. No. CF 0394 CV with Averaged Restoring Force

$$F_r = \pm 560\ (|x|)^{1.65} \tag{5.1}$$

The response curve for this case can be obtained from Eq. 2.38 by letting $\theta_1 = \pi/2$ and applying the approach of Klotter (1951). The response curve for $\zeta = 2$ ft/sec for the averaged symmetrical restoring force obtained in this way is presented in Fig. 5.26b. Only the maximum displacement of the moored boat in the positive x-direction is presented in this figure; the response curve of the maximum displacement in the minus x-direction would simply be a mirror image of the curve shown.

It is evident from a comparison of Figs. 5.26a and 5.26b that erroneous conclusions could be drawn about the response of this small boat if the restoring force had been considered to be symmetrical. Consider the case where the clearance between the boat and the slip is small. It would be possible to conclude, using an average restoring force system, that no damage would be expected due to waves of certain heights and periods. However, if like Fig. 5.26a, the response curve was asymmetrical there would be a distinct possibility that the boat would actually strike the dock.

5.5 Important Wave Periods for Sample Studied

To determine the important range of wave periods for this small sample of moored boats the periods of free oscillation have been replotted in Fig. 5.27 as a function of the ratio of the displaced weight to the restoring force. The magnitude of the restoring force used in the ratio W/F_r is the force which is predicted from Figs. 5.10 through 5.16 for certain displacements X_{max} and X_{min}. These displacements were obtained for given periods from the curves of free oscillation presented in Figs. 5.18 through 5.24. Therefore, Fig. 5.27 indicates the range of important wave periods for the free oscillations of the seven boats studied. Portion (a) of Fig. 5.27 shows these data for initial boat displacements in the positive x-direction and part (b) presents similar information on the free periods corresponding to initial displacements in the negative x-direction. It is seen that in both cases for a given value of W/F_r there is a maximum variation in the period among the curves of two to three fold. These curves are typically non-linear demonstrating, as for the response curves, that as the applied force increases the period of oscillation decreases. For small applied forces, approximately 0.1 to 0.01 times the displaced weight, the periods of oscillation vary from approximately 3 sec to 10 sec in both sets of data. For much larger forces, and corresponding tensions in the lines, the periods of free oscillation of these boats are all in the range of 2 sec to 3 sec, well below normal storm wave activity, but perhaps in the range of local wave activity, i.e., waves generated within the small boat harbors. This may not be too serious in the case of forced oscillations because of the effect of damping at these higher frequencies. Viscous damping usually has the effect of significantly reducing the magnitude of the oscillations without greatly affecting the periods of free oscillation. Therefore, at these small wave periods it would be expected that the magnitude of the motion and the resultant line tension would be reduced for the case of forced oscillations compared to the inviscid results which have been presented in Sections 4.5 and 5.4.

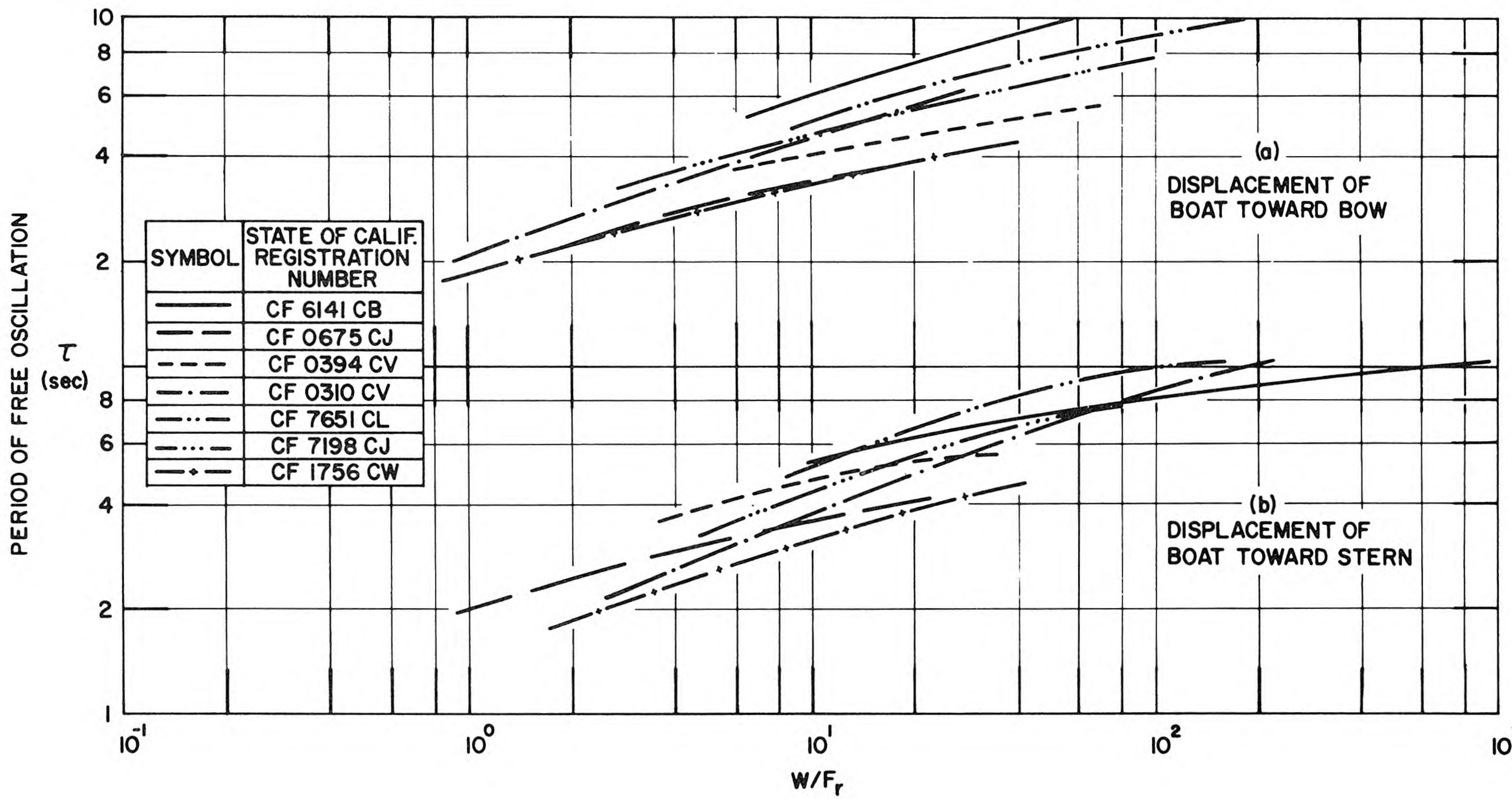

Fig. 5.27. Variation of Period of Free Oscillation with W/F_r

(a) Initial Displacement of Boat Toward Bow
(b) Initial Displacement of Boat Toward Stern

With Fig. 5.27 in mind, it appears that it would be possible to protect all of the boats investigated from surge damage due to storm waves by simply changing the mooring systems. For instance, the two extremes shown correspond to the smallest and the largest boats with the latter having periods of free oscillation which are a factor of two to three less than the former. This means that the curves which describe the forced oscillation of these two boats would compare in a similar way. Therefore, by stiffening the mooring system for the smaller boat its response curves would be shifted to a significantly smaller range of wave periods than is presently shown in Fig. 5.18. It is interesting to note from Fig. 5.27 that although the "backbone" curves for the three 25-foot boats (CF 0675 CJ, CF 0394 CV, CF 0310 CV) are not exactly the same, they do agree reasonably with one another. Again, differences among these three cases can be attributed to differences in the mooring systems which were discussed previously.

As has been emphasized in Section 4 and summarized in Table 4.3 the period of oscillation is very much a function of line slack for a given mooring system, i.e., large line slack results in large periods of free oscillation. Therefore, with this feature in mind, the mooring of small boats should probably be accomplished with a minimum of slack in the lines. The desired approach would be to reduce the periods of free oscillation of a boat by simply decreasing the permitted free travel thereby shifting the response curves for the forced oscillations to wave periods which are significantly less than the storm wave range (approximately 8 sec to 13 sec).

In summary, the important wave periods for this small sample of small boats moored in a harbor are generally much less than those for large vessels. Whereas large ships may have periods of free oscillation ranging from 20 sec to 90 sec (see Wilson (1967a)), the periods of free oscillation for this limited sample of small boats, corresponding to the assumptions discussed, were less than 10 sec. Hence, except for the case of larger pleasure boats, wave periods which could be expected to be important to small boat motions would be in and below the range of storm waves. Therefore, it would not appear to be necessary to test hydraulic models of small boat harbors for the same range of wave periods as harbors for large ships are tested.

6. LABORATORY STUDIES

In this section a series of experiments will be discussed which were conducted in the laboratory on small boat oscillations. The objective of the experiments was to determine the effect of a simplified mooring structure on the motion of the moored vessel.

6.1 Description of the Experimental Apparatus

The experimental equipment used is for the most part described by Raichlen (1965) and will only be summarized here. The wave basin is 21 inches deep with a working area 30 feet long by 12 feet wide. The wave generator is a pendulum type 12 feet wide located at one end of the basin and it is designed such that it can operate either as a piston or a flap wave generator. When operating as a flap for short period waves the imaginary hinge point is located close to the bottom of the flap. The wave machine is driven by two arms connected to independent Scotch yokes which are in turn driven through a pulley system by a variable speed motor. A maximum amplitude of ± 6 inches can be obtained through this arrangement and adjusted to within ± 0.0005 inches by means of dial gages located at opposite ends of the paddle. The motor drive is a 1-1/2 hp U.S. Varidrive motor with a 10:1 speed range and a continuous variation over this range. Wave periods from 0.34 seconds to 3.8 seconds can be obtained with this system. The motor has a greater power output than is actually needed for the waves which are generated and this combined with the "flywheel effect" of the Scotch yokes leads to a constant speed operation. The wave period is determined by a pulse counting technique; the pulses are generated by interrupting a light beam which is directed at a photo cell by a disc with 360 evenly spaced holes arranged in a circle around its outer edge. These voltage pulses which are produced by the electronics associated with the photo cell are counted by a Beckman/Berkeley Division Industrial Center Model 7361 over an interval of 10-seconds. For more details on this arrangement the interested reader is referred to Raichlen (1965).

The moored body used in this phase of the study was a rectangular parallelepiped 24 inches long, 6 inches wide, and 8 inches high. It was constructed of 1/4-inch lucite and it was built in such a way that it could be loaded with sand ballast to make it neutrally buoyant for a given depth of immersion (draft). Two aluminum leaf springs (0.09 inches thick and 2 inches wide) were used to represent the mooring system. These linear springs were arranged 2 inches apart and mounted to the body so that only longitudinal (surge) motions were permitted. The length of the springs, measured from a clamping device to the point of support on the body, could be varied to change the restoring force and hence the natural period of the body.

In these experiments it was desired to determine the effect of the proximity of a partially submerged body on the response of the simulated moored boat. The partially submerged body represents the flotation chambers to which is attached the floating slip surrounding the moored boat as illustrated diagrammatically in Fig. 4.4. The question was whether these chambers could affect the response of the small boat in a standing wave environment when the chambers were close to the moored boat. In the laboratory the flotation chamber or pontoon was simulated by a rectangular parallelepiped 6 inches wide and 12 feet long which spanned the working section of the wave tank and was mounted with its long axis parallel to the wave generator at a fixed distance from the back wall of the basin. A cross section of the major apparatus showing this arrangement is presented in Fig. 6.1. Both the draft of the pontoon and its location relative to the moored body could be varied.

The amplitude of the standing waves and the amplitude of the surge motion of the moored body were measured electrically using resistance wave gages and a linear variable differential transformer, respectively (see Raichlen (1965)).

6.2 Results and Discussion of Results

In order to study the effect on the response of the moored body of the proximity of the pontoon to the moored body the latter was kept at a fixed draft and distance from the backwall of the wave basin ($b = 4.0$ feet, $D = 0.376$ feet), and the distance between the moored body and the pontoon and the draft of the pontoon were varied (ℓ^*/L and D^*/D respectively). A summary of the experimental conditions is presented in Table 6.1. Experiments were conducted similar to those described by Raichlen (1965) in which wave amplitudes were measured at two locations along the backwall of the basin and the motion of the vessel in surge was measured for various periods of normally incident waves.

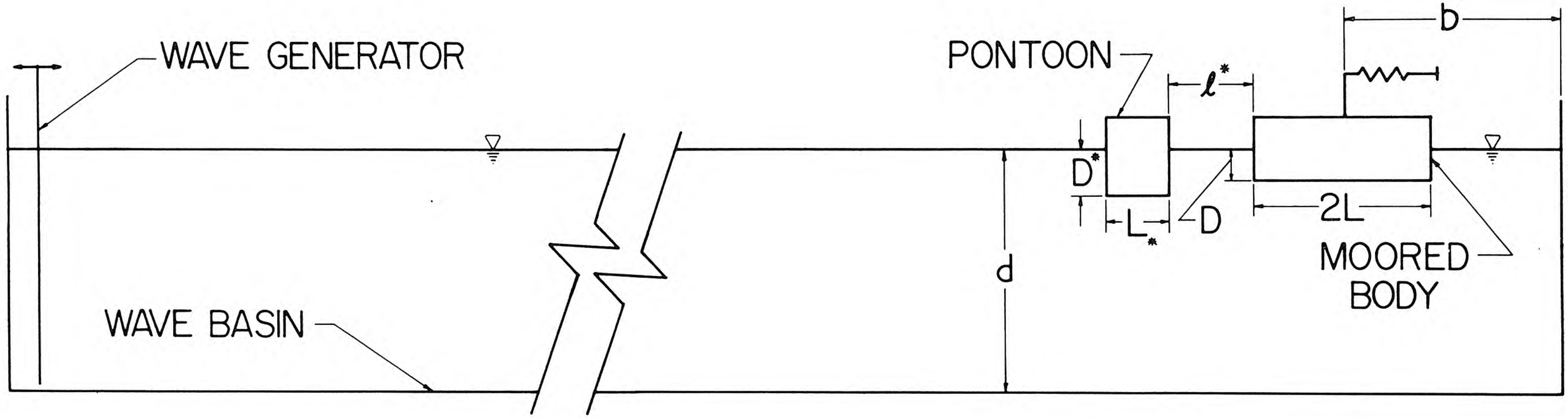

Fig. 6.1. Schematic Diagram of the Experimental Arrangement of Moored Block Body and Fixed Pontoon

Table 6.1. Experimental Conditions for Moored Block Body and Fixed Pontoon

Experiment	D^*/D	ℓ^*/L
15**	0	-
23	1.0	1.0
24	1.0	0.5
25	1.0	0.25
26	1.53	0.25

d = 1.0 foot	L = 0.5 feet
b = 4.0 feet	$2L^*$ = 2.0 feet
D = 0.376 feet	** See Raichlen (1965)

The results of this experimental study are presented in Figs. 6.2 and 6.3. Fig. 6.2 shows the response curve for the small moored body presented nondimensionally with the ordinate as the ratio of the amplitude and the abscissa as the ratio of the natural period of the body to the wave period. The theoretical curve shown in Fig. 6.2 is for the case without the pontoon ($D^*/D = 0$) in the absence of viscous effects (see Raichlen (1965)); the corresponding experimental data from previous studies are also included (Experiment 15, Raichlen (1965)). The data from two experiments are shown in Fig. 6.2 with the pontoon in place. In both experiments the pontoon had the same draft as the moored body; in one case the lee side of the pontoon was one-half a moored-body length from the seaward end of the body and in the second case this distance was reduced by one-half. (For the case of the 26 foot

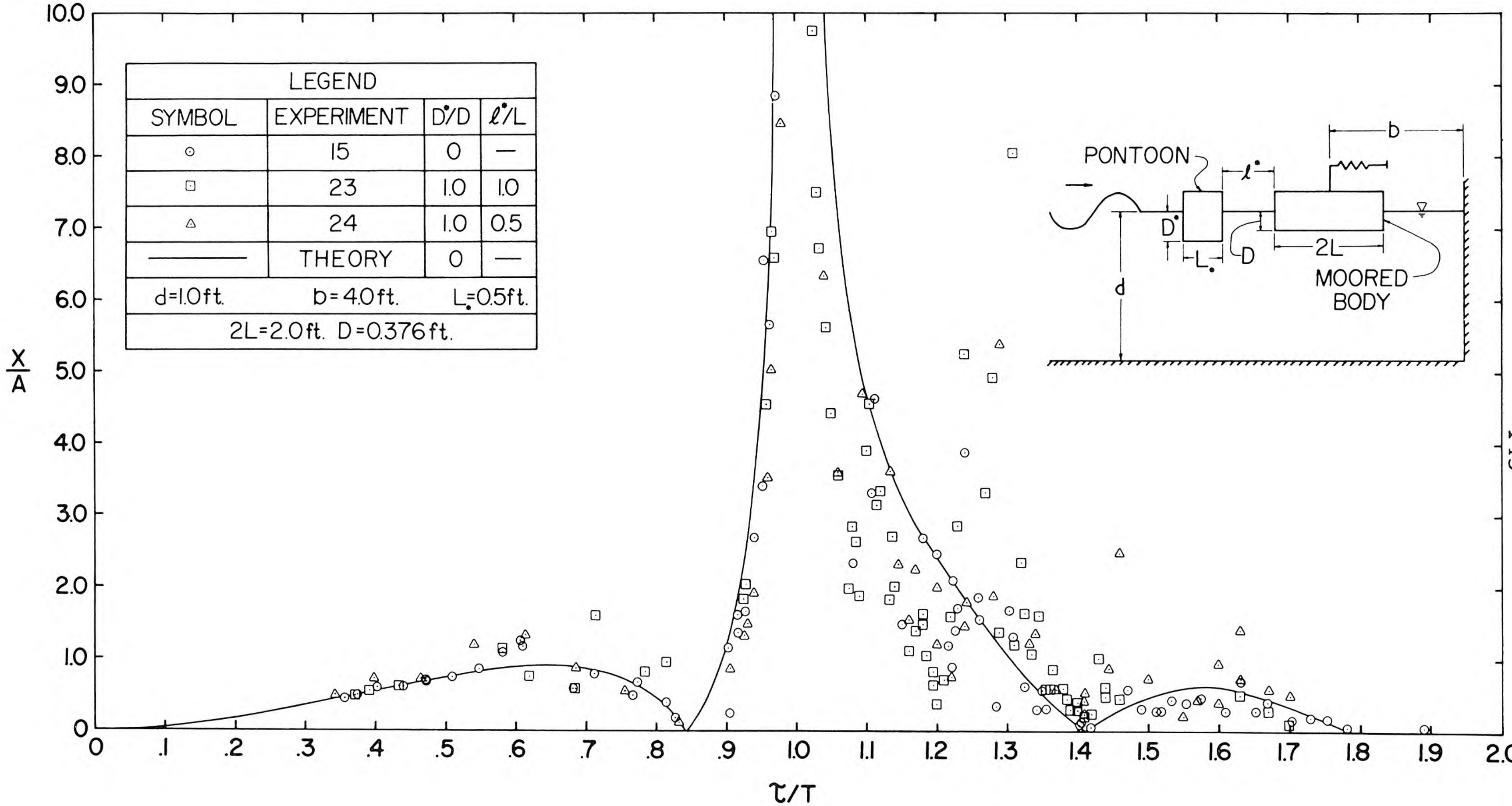

Fig. 6.2. Effect of Pontoon on Response of Moored Body
(Experiments 15, 23, 24)

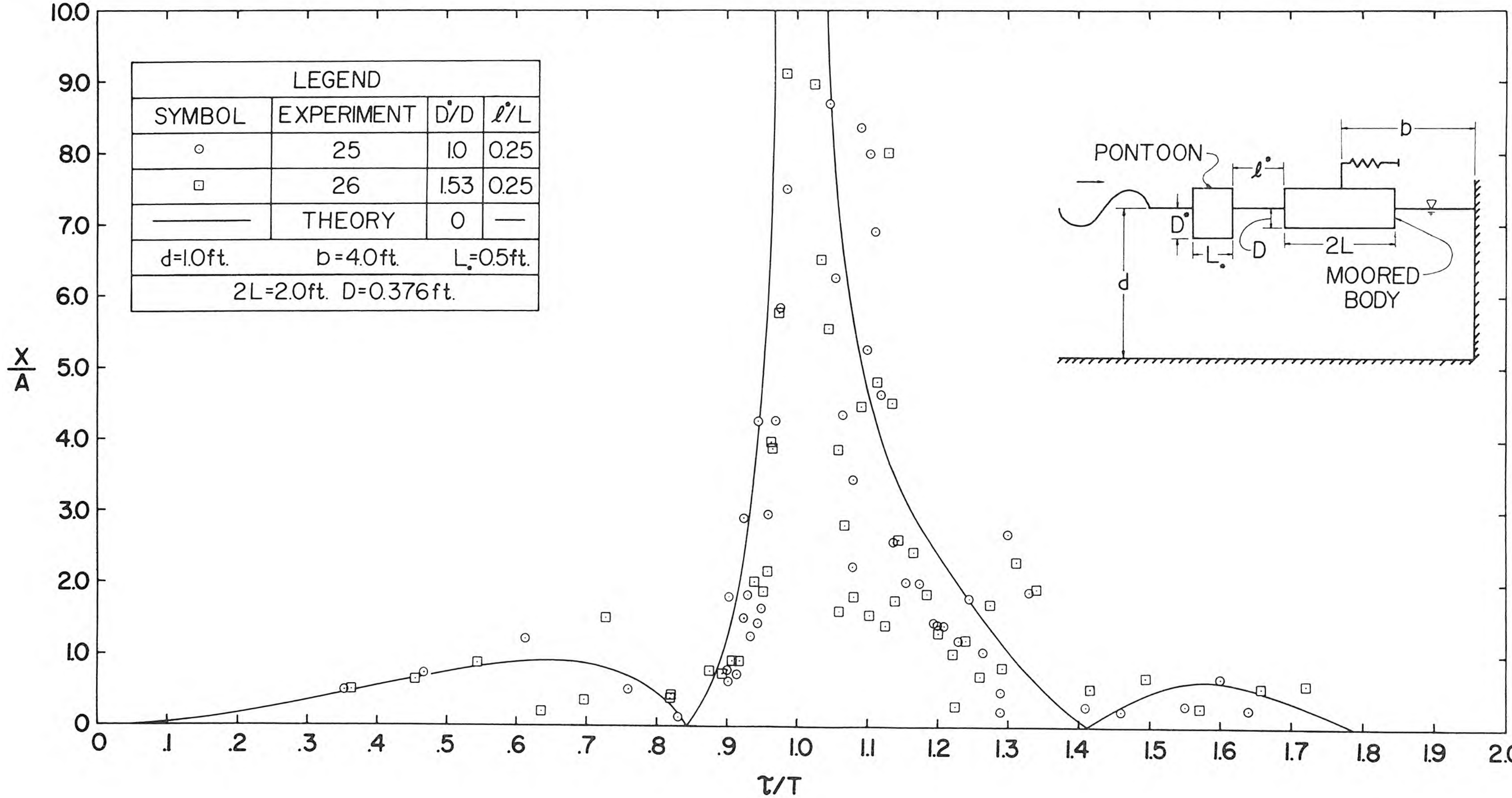

Fig. 6.3. Effect of Pontoon on Response of Moored Body
(Experiments 25, 26)

harbor boat discussed in Section 4, these experiments would correspond to distances of approximately 13 feet and 6.5 feet respectively.) For all cases the width of the pontoon was approximately one-quarter of the length of the body.

Originally it was considered possible that there would be a near field effect ofthe pontoon on the transmitted wave system which would modify the driving force on one end of the moored body. However, it is evident from Fig. 6.2 that as the distance decreases this effect is probably not significant (at least over the range of distances investigated). In fact except for the region $1.2 \leq \tau/T \leq 1.4$ the experimental data with the pontoon agrees as well with the theory for the case without the pontoon as does the corresponding experimental data for that arrangement.

Data are presented in Fig. 6.3 for the condition of a smaller clearance, $\ell^*/L = 0.25$, and for the same pontoon draft as the body, $D^*/D = 1$, (corresponding to a clearance of 3.3 feet in the example just mentioned). For that case the data show the same agreement with the theory for $D^*/D = 0$ as the data presented in Fig. 6.2. In addition, little difference exists between these data and those for a 50% larger draft ($D^*/D = 1.53$).

These experiments show that there is no significant effect on the normalized response of a moored body resulting from a pontoon or flotation chamber of the type depicted in the experiments located relatively close to it. The waves for the cases investigated range from nearly shallow water to deep water waves ($0.04 \leq d/\lambda/\leq 0.4$).

Therefore, the analytical method discussed in Section 2 is considered to be reasonably applicable to the case where the boat is moored in a "U" shaped floating slip where the slip is supported by flotation chambers with a draft of the order of that of the boat.

This certainly is not the complete picture, however, as the pontoon affects the amplitude of the waves within the portion of the basin leeward of the pontoon. It has been found that the transmission coefficient of this pontoon (defined as the ratio of the height of the wave at the back-wall of the basin with the pontoon in place to that at the same location without the pontoon) varies from 40% to 70% depending on wave period. Therefore, a preliminary conclusion which may be drawn from this phase of the study is that although the analytical method presented previously could be used to evaluate the response characteristics of small boats moored to a floating slip, an understanding of the transmission characteristics of such a slip-system is necessary to evaluate the actual amplitude of the motion of the moored body. Therefore, to be exact the value of A used in Eq. 2.17c which describes the forcing function should be the amplitude of the transmitted standing wave system. To use the value without transmissive effects would be somewhat conservative but it would yield the correct information on the important wave periods.

7. CONCLUSIONS AND RECOMMENDATIONS

The following major conclusions may be drawn from this study:

1. The theory which is developed to treat the surge motions of small boats which are moored asymmetrically with elastic-non-linear restraints appears adequate when the results are compared to experimental results of the free oscillations of a small moored boat.
2. A major difference between the mooring systems of small boats and large ships is the restoring forces which oppose the motions; for small boats these forces will probably be asymmetrical whereas relatively symmetrical restoring forces are expected for large moored ships. The asymmetrical restraint can lead to highly asymmetrical dynamic motions which can affect damage criteria developed for small craft harbors. For this reason, instead of a damage criterion based on the failure of lines, impact damage may be more important.
3. For forced motions of small boats the characteristics of the restoring force are extremely important. Relatively small line slack can significantly increase the resonant period of oscillation of a boat. The elastic characteristics of the mooring lines are equally important in determining the oscillations of small boats. It is recommended that additional studies be conducted to develop a rational method of predicting the elastic characteristics of

various ropes used in small craft moorings for a wide range of the line material and its condition.

4. An analytical study of the mooring dynamics of seven small boats has indicated that it is possible that the nature of the restoring force is more important in defining the periods of oscillation of these boats than is the displaced weight of the boat.

5. For the seven small boats studied the periods of free oscillation were less than about 10 sec. For forced oscillations of a small boat in surge the important wave periods would be increased by an amount dependent upon the magnitude of the forcing function and the permitted motion of the boat, but they would still probably remain in the range of those of storm waves.

6. For the boats investigated it was found that the ratio of the displaced weight of the boat to the restoring force for significant motions should be less than approximately 10 in order to limit the important wave periods for forced motions to values less than approximately 10 secs.

7. When it is possible to moor a small boat using taut and stiff lines this should be done to reduce the resonant periods well below 10 sec.

8. It should be emphasized that the forcing function which defines the dynamic response of any moored vessel in a standing wave environment is a function not only of the wave height but of the wave period and certain

dimensions of the mooring arrangement. Therefore, it is not reasonable to simply develop a criterion for the design of small craft harbors which relates only to minimizing the wave height; i.e., wave periods must also be considered along with pertinent system geometry.

9. Laboratory tests indicate that the flotation chambers associated with floating slips do not affect the character of the dynamic response of small vessels moored nearby. However, these chambers may act as a floating breakwater by reducing the absolute motion of a boat through the attenuation of transmitted wave energy. For this reason it is recommended that the design of floating slips be investigated in detail as one method of reducing wave-induced boat motions and potential boat damage.

10. The method of approach used in predicting the dynamic response of small moored boats has been described in some detail in this report. It is recommended that this method be applied to additional prototype data obtained from a large number of small moored boats in order to accumulate a significant amount of information on the mooring dynamics of small boats. It is possible that mooring standards can be developed, based

on such data, which could lead to a reduction in the cost of the construction of a small craft harbor. For instance, the motions of small boats may be reduced by the proper design of mooring systems and the introduction of certain mooring restrictions rather than by drastically reducing the wave energy which could enter the small boat harbor.

LIST OF SYMBOLS

A	Amplitude of standing wave (ft)
a_1, a_2, a_3	Coefficients used in polynomial representation of restoring force, $x > 0$
B	Beam of boat (ft)
b	Distance from reflecting surface to center of boat (ft)
C_{D_x}	Drag coefficient in x-direction
C_M	Virtual mass coefficient in surge
D	Draft of boat (ft)
D^*	Draft of pontoon (ft)
d	Depth of water (ft)
F_r	Restoring force (lbs)
f	Horizontal distance from mooring cleat on dock to first point of contact on boat (ft)
g	Acceleration of gravity (32.2 ft/sec^2)
K	Linear spring constant lbs/ft
k	Wave number $= 2\pi/\lambda$
L	Half-length of boat (ft)
L_*	Width of pontoon (ft)
ℓ	Distance from mooring cleat on dock to first point of contact on boat (ft)
ℓ_1	Length of line on boat running from first point of contact on boat to cleat or bit on boat (ft)
ℓ^*	Distance between front of moored body and rear face of pontoon (ft)

LIST OF SYMBOLS (cont'd)

M	Mass of boat (slugs)
M'_x	Added hydrodynamic mass in surge (slugs)
p	Pressure (lbs/ft^2)
r_1, r_2, r_3	Coefficients used in polynomial representation of restoring force, $x < 0$
s	Clearance from bottom to bottom of boat (ft)
T	Wave period (sec)
T^*	Line tension (lbs)
$T^*_{BRK.}$	Average breaking strength of line (lbs)
t	Elapsed time (sec)
U	Water particle velocity averaged over displaced volume (ft/sec)
$\dot{U}$	Water particle acceleration over displaced volume (ft/sec^2)
u	Water particle velocity (ft/sec)
$\dot{u}$	Water particle acceleration (ft/sec^2)
W	Displaced weight (lbs)
X	Amplitude of boat's motion about mean position (ft)
X_{max}	Maximum displacement of boat in the + x-direction from the at-rest-position (ft)
X_{min}	Maximum displacement of boat in the - x-direction from the at-rest-position (ft)
x	Coordinate measured from center of boat positive in direction of bow (ft)

LIST OF SYMBOLS (cont'd)

y_o	Lateral distance from mooring cleat on dock to first point of contact on boat (ft)
z	Coordinate measured from water surface positive upward (ft)
z_o	Vertical distance from mooring cleat on dock to first point of contact on boat (ft)
α	Angle describing inclination of line
β	Angle describing inclination of line
γ	Specific weight of water (fresh water = 62.4 lbs/ft^3 sea water = 64 lbs/ft^3)
Δ_f	Free travel of boat (ft)
$\Delta\ell$	Slack in line (ft)
δ	Distance between mean position of boat's motion and at-rest-position (ft)
ϵ	Line strain
ζ	Wave function ft/sec (fps)
η	Distance from MWL to water surface (ft)
θ	σt
λ	Wave length (ft)
π	3.14159
ρ	Density of water = γ/g (slugs/ft^3)
σ	Circular wave frequency = $2\pi/T$ (rad/sec)

LIST OF SYMBOLS (cont'd)

τ	Period of free-oscillation (sec)
Φ	Velocity potential (ft^2/sec)
φ	Phase angle (radians)

LIST OF REFERENCES

Chaney, C. A., "Marinas: Recommendations for Design, Construction, and Maintenance", National Association of Engine and Boat Manufacturers, Inc., 1961.

Den Hartog, J. P., "Mechanical Vibrations", McGraw Hill Book Co., 4th ed., 1956, pp. 353-363.

E. I. DuPont de Nemours & Co. (Inc.), "The Effect of Loading on the Extension and Recovery of Ropes of Nylon and Dacron", Bulletin X-92, Wilmington, Delaware, 1958.

Kilner, F. A., "Model Tests on the Motion of Moored Ships Placed in Long Waves", Proc. of 7th Conf. on Coastal Engr., The Hague, 1960.

Klotter, K., "Non-Linear Vibration Problems Treated by the Averaging Method of W. Ritz", Proc. of 1st U.S. Nat. Cong. of Appl. Mech., Chicago, Ill., 1951.

Raichlen, F., "Wave-Induced Oscillations of Small Moored Vessels", Report KH-R-10, W.M. Keck Lab. of Hydraulics & Water Resources, Calif. Inst. of Tech., Pasadena, Calif., Oct. 1965.

Tubbs Great Western Cordage, "Superlene and Duolene Rope", Publ. S-19, San Francisco, Calif.

Wilson, B. W., "The Energy Problem in the Mooring of Ships Exposed to Waves", Proc. of Princeton Conf. on Berthing and Cargo Handling in Exposed Locations, Princeton Univ., New Jersey, Oct. 1958.

Wilson, B. W., "The Threshold of Surge Damage for Moored Ships", Proc. Instn. Civ. Engrs., 38, Sept. 1967(a).

Wilson, B. W., "Elastic Characteristics of Moorings", Journ. of Waterways and Harbors Div., ASCE, Nov. 1967(b).

SUPPLEMENTARY REFERENCES

Abramson, H. N. and Wilson, B. W., "A Further Analysis of the Longitudinal Response of Moored Vessels to Sea Oscillations", Proceedings of the Joint Midwestern Conference on Solid and Fluid Mechanics, Purdue University, Indiana, Sept. 1955.

Browne, A. D., Moullin, E. B. and Perkins, A. J., "The Added Mass of Prisms Floating in Water", Proc. Cambr. Phil. Soc., v. 26, 1929-30, pp. 258-272.

Dunham, J. W., "Design Considerations for California Marinas", Trans. ASCE, Vol. 127, Part IV, 1962, pp. 131-162.

Knapp, R. T., "Wave Produced Motion of Moored Ships", Proceedings of the Second Conference on Coastal Engineering, Houston, 1951.

Koch, J. J., "Eine experimentale Methode zur Bestimmung der Reduzierten Masse des Mitschwingenden Wassers bei Schiffschwingsingen", Ingenieur-Archiv, band 4, heft 2, 1933, pp. 103-109.

Korvin-Kroukovsky, B. V., "Theory of Seakeeping", The Society of Naval Architects and Marine Engineers, New York, N.Y., 1961.

Lee, C. E., "Small-Craft Harbor Problems", Journal of the Waterways and Harbors Division, ASCE, Aug. 1964.

O'Brien, J. T. and Kuchenreuther, D. I., "Forces Induced on a Large Vessel by Surge", Journal of the Waterways and Harbors Division, ASCE, Paper 1571, March 1958.

O'Brien, J. T. and Muga, B. J., "Sea Tests of a Spread-Moored Landing Craft", U.S. Naval Civil Engineering Laboratory, Technical Report R-268, June 1964.

Ransford, G. D., "A Wave Machine of Novel Type", Proceedings of Third Congress of International Association for Hydraulic Research, Grenoble, 1949.

Russell, R. C. H., "A Study of the Movement of Moored Ships Subjected to Wave Action", Proceedings of the Institution of Civil Engineers, Vol. 12, April 1959.

Wendel, K., "Hydrodynamic Masses and Hydrodynamic Moments of Inertia", The David Taylor Model Basin, Translation 260, July 1956.

Wiegel, R. L., Dilley, R. A., and Williams, J. B., "Model Study of Mooring Forces of Docked Ships", Journal of Waterways and Harbors Division, ASCE, Vol. 85, WW2, 1959.

Wilson, B. W., "Ship Response to Range Action in Harbor Basins", Trans. ASCE, Vol. 116, 1951, pp. 1129-1127.

Wilson, B.W., "Case of the Critical Surging of a Moored Ship", Trans. ASCE, Vol. 126, Part IV, 1961.

Yu, Y. S. and Ursell, F., "Surface Waves Generated by an Oscillating Circular Cylinder on Water of Finite Depth: Theory and Experiment", Journal of Fluid Mechanics, Vol. 11, Part 4, 1961.

ACKNOWLEDGMENTS

This investigation was sponsored by the U. S. Army Corps of Engineers under Contract DA-22-076-CIVENG-64-11. The author would like to acknowledge the valuable discussions with Professor Vito A. Vanoni throughout the course of this investigation and with Professors Wilfred D. Iwan and Paul C. Jennings during early phases of this study.

Elton F. Daly, Laboratory and Shop Supervisor, contributed significantly throughout the prototype and laboratory studies through his ingenuity and assistance along with Robert Greenway, Senior Experimental Mechanic. James Murray and William Pence assisted in various capacities during the prototype studies. Without the talent and aid of Carl Eastvedt, Senior Photographer, the prototype study would have been much more difficult to conduct.

The assistance of both the U. S. Army Corps of Engineers, Los Angeles District and the Department of Small Craft Harbors, County of Los Angeles, in arranging and assisting with the prototype studies is appreciated. In particular the author appreciates the cooperation extended by William Herron, Chief, Coastal Engineering Branch, U. S. Army Engineer District, Los Angeles, Corps of Engineers, and James Quinn of the Department of Small Craft Harbors, County of Los Angeles.

The drawings in this report were made by Carl Green, Laboratory Technician. The writer wishes to thank Patricia Rankin for her invaluable assistance in typing and assembling this report.

DISTRIBUTION LIST

Office	No. of Copies	Remarks
OCE (ENGAS-I)	2	
OCE (ENGCW)	2	
OCE (ENGSA)	1	
OCE (ENGME)	1	
Bd of Engrs for Rivers & Harbors	1	
Engr Center, Fort Belvoir	1	
Engr School Library Fort Belvoir	1	
CERC	2	
CERB	1	ATTN: Dr. Ippen
	1	ATTN: Dean O'Brien
Supv N. Y. Harbor	1	
LMVD	1	DE
Memphis	1	ATTN: Tech Library
New Orleans	1	DE
	1	ATTN: Hydraulic Br, Engrg Div
St. Louis		Abstract: DE
		Abstract: Chief, Hydraulics Br (LMSED-H)
Vicksburg	1	ATTN: Hydraulics Branch
MRD	4	ATTN: Off, Administrative Services (Library)
Kansas City	1	ATTN: District Library
NAD	1	DE
	1	ATTN: Engineering Division
	1	ATTN: Mr. O. F. Reyholec
	1	ATTN: Mr. A. G. Distefano
	1	ATTN: Planning Division
	1	ATTN: Civil Works Br, Constr-Oper Division
	1	ATTN: Mr. Morris Colen
Baltimore	1	DE
	1	ATTN: Engineering Division
	1	ATTN: Project Planning Branch

Office	No. of Copies	Remarks
NAD (Continued)		
New York	1	DE
	1	ATTN: Engineering Division
	1	ATTN: Basin & Project Planning Branch
	1	ATTN: Chief, Beach Erosion & Hurricane Sec
Norfolk	1	DE
	1	ATTN: Engineering Division
Philadelphia	1	DE
	1	ATTN: NAPEN-H, Engineering Division
	1	ATTN: NAPEN-D, Engineering Division
NCD	1	ATTN: Library
	1	ATTN: Hydraulics Branch
	1	ATTN: Mr. R. E. Emmenegger
Buffalo	1	ATTN: Engineering Division
	1	ATTN: Mr. T. V. Joyce
Chicago	1	ATTN: Engineering Division
	1	ATTN: Project & Basin Planning Branch
	1	ATTN: Operations Division
Detroit	1	DE
	1	ATTN: Library
Rock Island	1	DE
St. Paul	1	DE
	1	ATTN: Engineering Division
Lake Survey	3	ATTN: Technical Library
NED	1	ATTN: Engineering Division
	1	ATTN: Planning Branch
NPD	1	ATTN: Planning Division
	1	ATTN: Division Hydraulic Laboratory
	1	ATTN: Geology, Soils & Materials Branch
	1	ATTN: Water Control Branch
Alaska	1	DE
	1	ATTN: District Library
	1	ATTN: Hydraulics Design Section
	1	ATTN: Planning & Reports Branch
	1	ATTN: Foundations & Materials Branch
Portland	1	ATTN: District Library
Seattle	1	DE
	1	ATTN: Engineering Division
	1	ATTN: Planning Branch

Office	No. of Copies	Remarks
NPD (Continued)		
Walla Walla	1	ATTN: Engineering Division
	1	ATTN: Planning & Reports Branch
ORD	1	DE
	1	ATTN: Mr. W. H. Browne, Jr.
Huntington	1	ATTN: Library
	1	ATTN: Hydraulics Branch
Louisville	1	ATTN: Hydraulics Branch
Nashville	1	ATTN: Hydraulics Br, Engrg Division
Pittsburgh	1	ATTN: Engineering Div Tech Library
POD	1	ATTN: Tech Engrg Br, PODGB
	1	ATTN: Mr. W. D. Darling
Honolulu	1	ATTN: District Library
	1	ATTN: Engineering Division
	1	ATTN: Divil Works Branch
	1	ATTN: Mr. Clarence Fujji
SAD	1	ATTN: Engineering Division
	1	ATTN: Planning Division
Charleston	1	DE
	1	ATTN: Engineering Division
	1	ATTN: Coastal Engineering Branch
Jacksonville	1	ATTN: Project Planning Branch
	1	ATTN: Chief, Canal Hydraulic Section
	1	ATTN: Engineering Division
	1	ATTN: Design Branch
	1	ATTN: Water Management Section
	1	ATTN: Library
Mobile	1	DE
	1	ATTN: SAMEN-DV
	1	ATTN: SAMEN-P
Savannah	1	DE
	2	ATTN: Library
Wilmington	1	DE
	1	ATTN: Engineering Division
	1	ATTN: Planning, Report, & Program Branch
SPD	1	ATTN: Chief, Technical Engineering Branch
	1	ATTN: Planning Division
	1	ATTN: Mr. O. T. Magoon
	1	ATTN: Mr. Olin Weymouth

Office	No. of Copies	Remarks
SPD (Continued)		
Los Angeles	1	ATTN: Mr. W. J. Herron, Jr.
	1	ATTN: Library
	1	ATTN: Engineering Division
	1	ATTN: Coastal Engineering Branch
	1	ATTN: Mr. G. D. Ward
		Abstracts: Mr. Edward Koehm, Mr. Robert S. Perkins, Mr. Albert P. Gildea, Project Planning Branch
Sacramento	2	District Librarian
San Francisco	4	ATTN: Library
	1	ATTN: Mr. O. J. Pietsch
	1	ATTN: Mr. G. P. Riley
SWD	1	DE
	2	ATTN: Library
Albuquerque	2	ATTN: Engineering Div Library
		Abstract: Engineering Division Library
		Abstract: Mr. Lewis C. Slack
Fort Worth	1	ATTN: Librarian
Galveston	1	DE
	1	ATTN: Librarian
	1	ATTN: Engineering Division
	1	ATTN: Project Planning Branch
Little Rock	1	DE
Tulsa	1	DE

Automatic:

Engineering Societies Library, New York, N. Y.	1
Library, Div of Public Doc, U. S. Govt Print. Office, Washington, D.C.	1
Library of Congress, Doc Expd Proj, Washington, D. C.	3
COL C. T. Newton	1
Coastal Engrg Research Center, ATTN: Mrs. Bennie F. Maddox, Librarian	1
Prof. J. W. Johnson, Univ of Calif., Berkeley, Calif.	1
Mr. R. Q. Palmer, Suite 2311, 1350 Ala Moana, Honolulu, Hawaii	1

Exchange Basis:

HOUILLE BLANCHE, Grenoble, France (ENG-63)	1
The Library, Nat'l Res Council, Ottawa, Canada	1
Librarian, Ministry of Technology at Kingsgate House, London (ENG-46)	1
The Inst of Civil Engineers, London, England (ENG-47)	2
Inst of Engineers, Sidney, Australia (ENG-162)	1
APPLIED MECHANICS REVIEWS, San Antonio, Tex.	2
Dept of Civil Engineering, Univ of Arizona, Tuscon, Ariz.	1
Dr. Rex K. Rainer, Auburn Univ, Auburn, Ala.	1
Bureau of Reclamation	1

Exchange Basis: (Continued)

Scripps Inst of Oceanography, Univ of Calif., LaJolla, Calif.	1
Engrg Lib, Univ of Calif., Berkeley, Calif.	1
Central Records Library, Dept of Water Resources, Sacramento, Calif.	1
W. M. Keck Lab of Hydraulics & Water Resources, Calif. Inst of Tech, Pasadena, Calif.	1
Case Inst of Tech, Cleveland, Ohio	1
Central Serial Record Dept, Cornell Univ Lib, Ithaca, N. Y.	1
Engrg & Ind Experi Sta, Univ of Florida, Gainesville, Fla.	1
Price Gilbert Memorial Library, Georgia Inst of Tech, Atlanta, Ga.	1
Gordon McKay Library, Cambridge, Mass.	1
Gift & Exchange Division, Univ of Ill. Library, Urbana, Ill.	1
Library, Iowa State Univ of Science & Tech, Ames, Iowa	1
Engrg Experi Sta, Kansas State Univ of Agric & Applied Science, Manhattan, Kans.	1
Documents Room, Univ Lib, Univ of Kansas, Lawrence, Kans.	1
Fritz Engineering Lab, Lehigh Univ, Bethlehem, Pa.	1
Hydrodynamics Laboratory, MIT, Cambridge, Mass.	1
University of Arkansas, Fayetteville, Ark.	1
Mr. Robert T. Freese, Univ of Michigan, Ann Arbor, Mich.	1
Engineering & Ind Research Sta, State College, Miss.	1
College of Engineering, Univ of Missouri, Columbia, Mo.	1
Univ of Missouri, School of Mines & Metallurgy, Rolla, Mo.	1
North Carolina State College, Raleigh, N. C.	1
Dept of Civil Engrg, Technological Inst, Northwestern Univ, Evanston, Ill.	1
Main Library, Ohio State Univ, Columbus, Ohio	1
Engrg Experi Sta, Oregon State Univ, Corvallis, Oreg.	1
Dept of Oceanography, Oregon State Univ, Corvallis, Oreg.	1
New York University, ATTN: Engrg Lib, University Heights, Bronx, N. Y.	1
McGill University, Montreal, Quebec, Canada (ENG-271)	1
Engrg Library, Pennsylvania State Univ, University Park, Pa.	1
Periodicals Checking Files, Purdue Univ Libraries, Lafayette, Ind.	1
Tennessee Valley Authority	1
Prof. John B. Herbich, Department of Civil Engineering, Texas A&M University, College Station, Tex.	1
Office of Engineering Research, Publications, Univ of Washington, Seattle, Wash.	1
Albrook Hydraulic Lab, Washington State Univ, Pullman, Wash.	1
Engrg Lib, Univ of Wisconsin, Madison, Wis.	1
Engineering Library, Stanford University, Stanford, Calif.	1
Serials Acquisitions, Univ of Iowa Libraries, Iowa City, Iowa	1
Lorenz G. Straub Memorial Library, Minneapolis, Minn.	1
Mr. Duncan Hay, Regional Coastal Engr, Dept of Pub Wks of Canada, Vancouver 5, B. C., Canada (ENG-318)	1
Director, Public Works Research Inst, Ministry of Constr, Bunkyo-ku, Tokyo, Japan (ENG-324)	1

Abstract of Report:
Princeton University River & Harbor Library, Princeton, N. J.
Duke University Library, Durham, N. C.
Princeton University Library, Princeton, N. J.
Louisiana State University Library, Baton Rouge, La.
The Johns Hopkins University Library, Baltimore, Md.
University of Kansas Libraries, Lawrence, Kans.
Laboratorio Nacional de Engenharia Civil, Lisboa, Portugal
Dept of Civil Engrg, Univ of Tokyo, Bunkyo-ku, Japan
Mr. Shigematsu Suzuki, Civil Engrg Res Inst, Hokkaido Development Bureau, Nakanoshima, Sappo, Japan
Commandant, USAREUR Engineer-Ordnance School, APO New York 09172
Mr. J. C. Harrold, 4216 Pennsylvania Ave., Apt. 2, Kettering, Ohio
Water Information Center, Inc., Water Research Bldg, 44 Sintsink Drive, East, Port Washington, N. Y.
Duke University, College of Engineering Library, Durham, N. C.
Serials Record, Pennsylvania State University, University Park, Pa.

Send with Bill:
Librarian, Dept of Public Works, Sydney, Australia 1

Announcement of Availability by Tech Liaison Branch:
CIVIL ENGINEERING
THE MILITARY ENGINEER
ENGINEERING NEWS-RECORD

Unclassified

Security Classification

DOCUMENT CONTROL DATA - R & D

(Security classification of title, body of abstract and indexing annotation must be entered when the overall report is classified)

1. ORIGINATING ACTIVITY (Corporate author) W. M. Keck Laboratory California Institute of Technology Pasadena, California	2a. REPORT SECURITY CLASSIFICATION Unclassified 2b. GROUP

3. REPORT TITLE

MOTIONS OF SMALL BOATS MOORED IN STANDING WAVES

4. DESCRIPTIVE NOTES (Type of report and inclusive dates)

Final report

5. AUTHOR(S) (First name, middle initial, last name)

Fredric Raichlen

6. REPORT DATE	7a. TOTAL NO. OF PAGES	7b. NO. OF REFS
August 1968	166	26

8a. CONTRACT OR GRANT NO. DA-22-079-CIVENG-64-11	9a. ORIGINATOR'S REPORT NUMBER(S) KH-R-17
b. PROJECT NO.	
c.	9b. OTHER REPORT NO(S) (Any other numbers that may be assigned this report) U. S. Army Engineer Waterways Experiment Station Contract Report H-68-2
d.	

10. DISTRIBUTION STATEMENT

This document has been approved for public release and sale; its distribution is unlimited.

11. SUPPLEMENTARY NOTES Prepared for U. S. Army Engineer Waterways Experiment Station, CE, Vicksburg, Miss.	12. SPONSORING MILITARY ACTIVITY Office, Chief of Engineers Washington, D. C.

13. ABSTRACT This study was conducted to determine the dynamic characteristics of small boats moored with non-linear-elastic lines in an asymmetrical manner. The motions being considered are surge motions where the moored boat is allowed to move either in the direction of the bow or the stern, but not in other coordinate directions. An analytical model is proposed where the small boat is simulated by a block-body which is moored asymmetrically to a fixed dock. A method is developed from which the non-linear restoring forces and the dynamic response of the boat in surge can be obtained. The restoring force which is associated with the boat displacement is defined by the material, condition, and dimensions of the lines and the mooring geometry. From those results, an approximation to the restoring force is made so that a closed solution to the problem is possible. The periods of free oscillation determined by this method are compared with the results of some experiments conducted on a 26-foot boat with a displaced weight of approximately 7000 lbs. The experiments were performed using this small boat moored under different conditions: all lines taut, 4 inches slack in all lines, and 8 inches slack in all lines. These results compared favorably with the analytical results. The response of seven small boats of various displaced weights was determined analytically to evaluate the range of important wave periods for this sample. The mooring dimensions of these boats were measured in situ and the theoretical approach developed was applied. The results indicate, for the samples considered, that the important range of periods of forced oscillation for excessive motions of these boats in surge was less than 10 secs. If stiff mooring systems had been employed for all of these boats the important wave period range for these motions could probably be reduced further. Due to the different mooring systems used, (Continued)

DD FORM 1 NOV 65 1473 REPLACES DD FORM 1473, 1 JAN 64, WHICH IS OBSOLETE FOR ARMY USE.

Unclassified

Security Classification

Unclassified

Security Classification

14. KEY WORDS	LINK A		LINK B		LINK C	
	ROLE	WT	ROLE	WT	ROLE	WT
Boats						
Mooring						
Standing waves						
Surges						
Water waves						

13. ABSTRACT (Continued)

the response curves for some of the small boats were highly asymmetrical indicating the possibility of much greater motions in one direction than in another under the action of a periodic symmetrical force. A limited series of experiments were conducted to determine the effect of the proximity of flotation chambers which are used on some floating slips on the response of the moored boat. It was found that these chambers, as simulated in the laboratory, did not have a significant effect on the dynamic characteristics of the moored boat. However, they did act as floating breakwaters thereby reducing the transmitted wave energy.

Unclassified

Security Classification